Perturbative computations for the LHC

Kirill Melnikov
Johns Hopkins University

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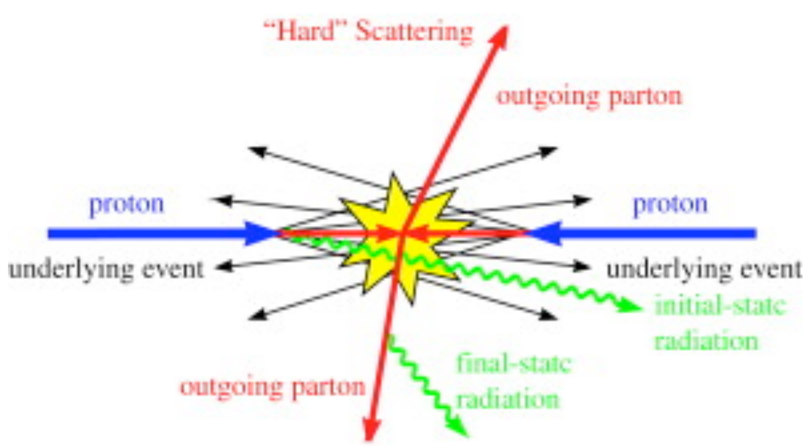
Perturbative computations for the LHC

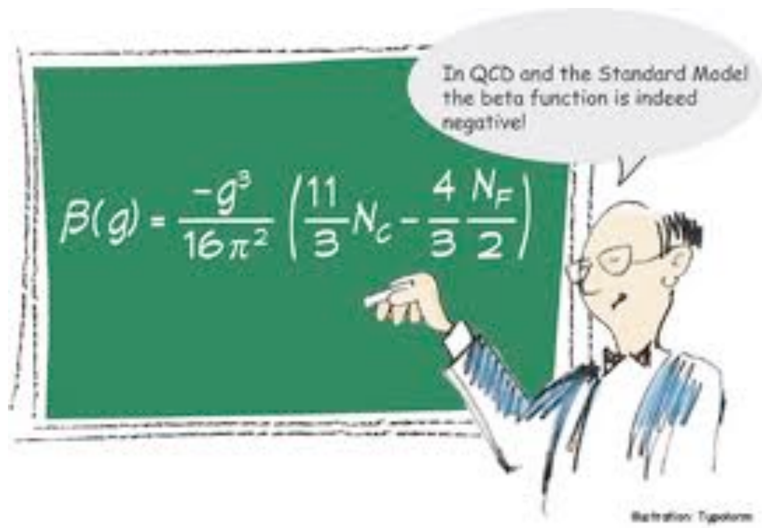
LHC looks for heavy physics beyond the Standard Model in hard collisions. Such collisions occur at small distances where asymptotic freedom of QCD is at work.

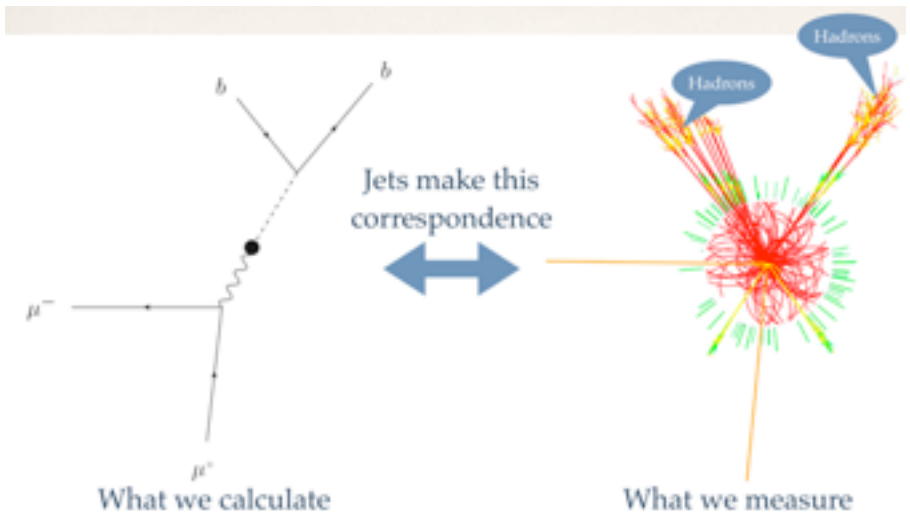
Since confinement is soft, properties of jets are not affected by non-perturbative phenomena. For this reason, jets provide a reliable probe of short-distance physics that can be used at large distances.

Factorization theorems imply that, for the purpose of describing hard inelastic collisions, protons can be treated as beams of quarks and gluons.

$$\mathcal{L}_{\rm QCD} = \bar{q}(i\gamma^\mu \partial_\mu - g_s \gamma^\mu A_\mu) q - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$





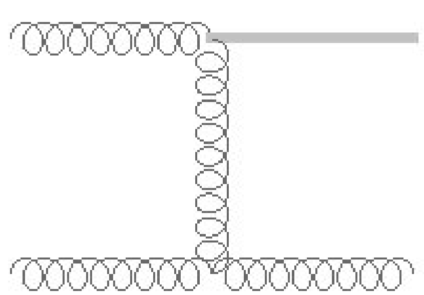


Perturbation theory for quark-gluon S-matrix

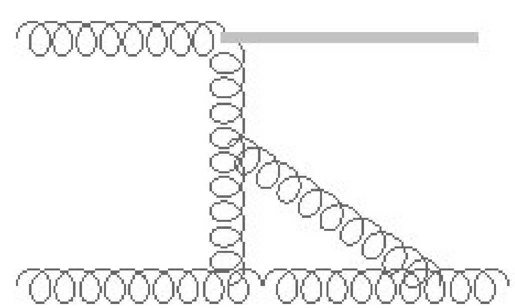
To describe collisions of quark-gluon beams, we use conventional perturbation theory where a small parameter is the QCD coupling constant. Our goal is to compute jet cross-sections for fixed number of jets and arbitrary number of colorless particles (Z,W,H, etc.)

We start with identifying each jet with a single parton; this defines the leading order approximation. We improve on it by adding both elastic (loops) and inelastic (additional gluons) corrections to the leading order approximation.

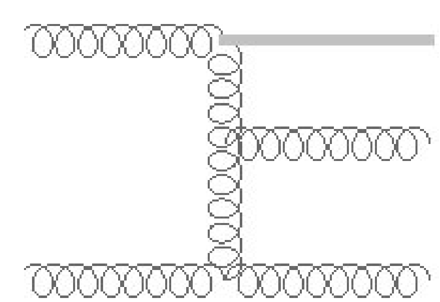
The need to combine elastic and inelastic contributions is related to the absence of mass gap in pQCD and the ensuing infra-red and collinear divergences in processes with fixed parton multiplicities. Kinoshita-Lee-Nauenberg theorem ensures their cancellation for properly-defined observables.



Leading order process



Elastic correction

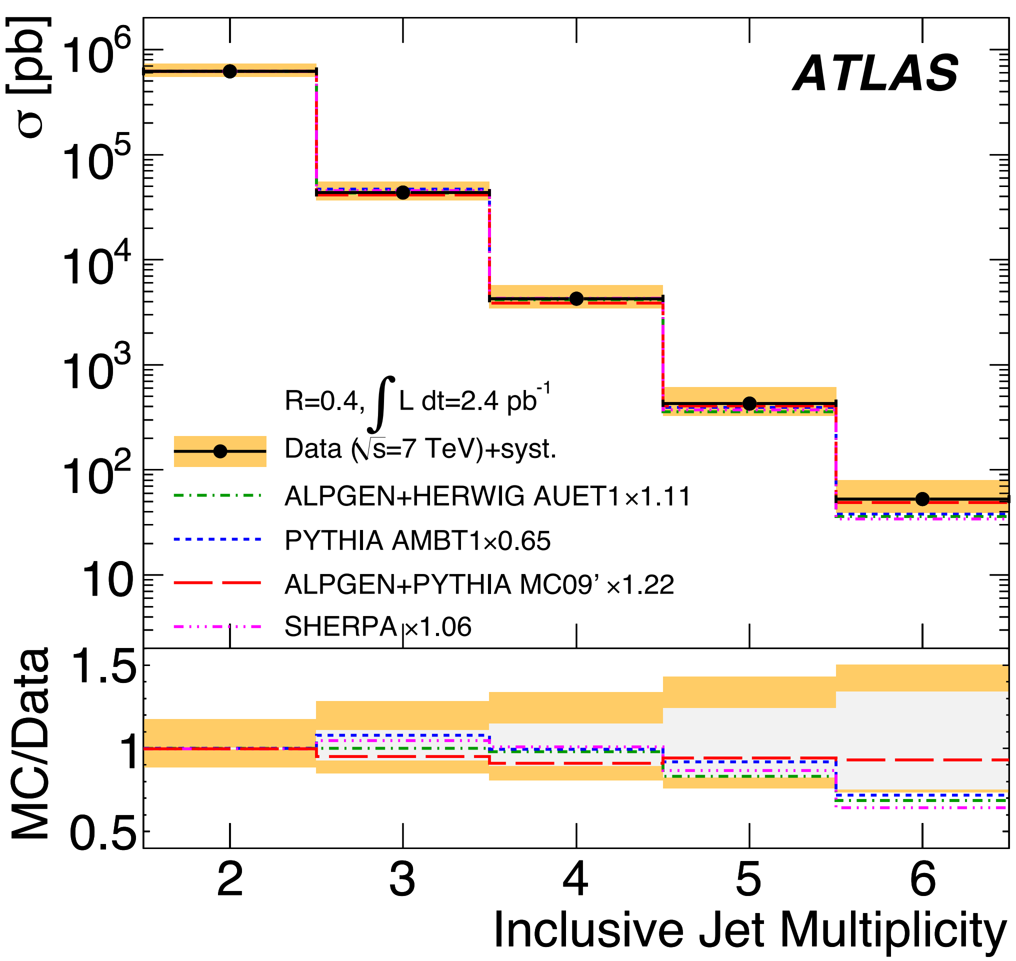


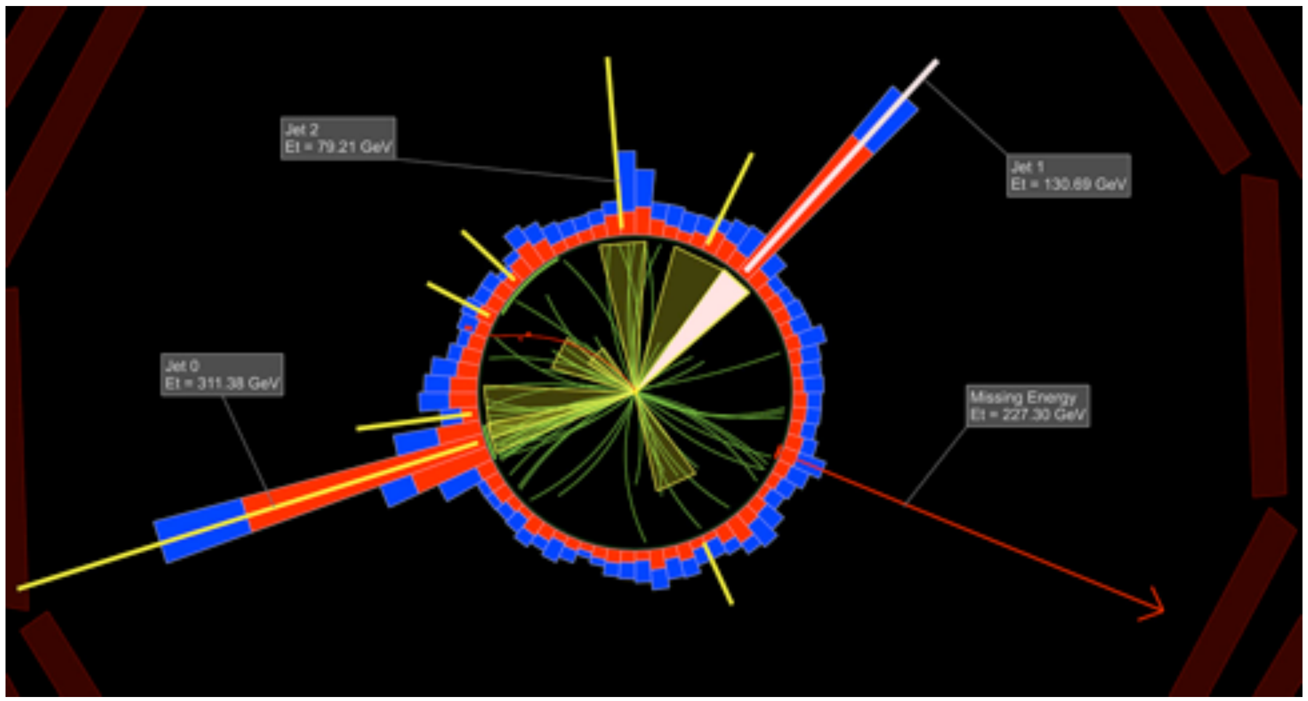
Inelastic correction

Perturbation theory for quark-gluon S-matrix

LHC is clearly a multi-jet collider. Processes with 5-6 jets are seen with tiny luminosity, 2.4/pb! Clearly processes with 9-10 jets in the final state will be seen with the luminosity that is available now.

To describe these processes, we must have a way to handle such high-multiplicity final states in perturbative QCD.





Perturbation theory for quark-gluon S-matrix

Two main approaches:

1) Parton showers / resummations: additional kinematic approximations (soft or collinear radiation) enable all-orders construction of the (approximate) S-matrix.

Talk by B. Webber

2) Canonical application of perturbation theory: all terms of the same order are calculated and combined, and no kinematic approximations are used. This talk

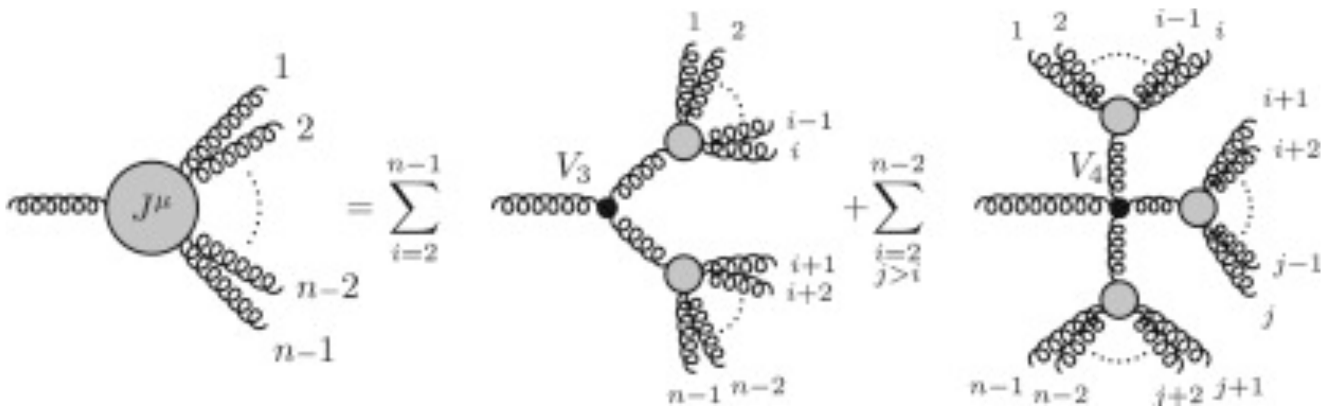
The success and relevance of the second approach for the LHC physics depends on how far we can push fixed-order perturbation theory in terms of jet multiplicities and the order of the perturbative expansion.

We know now that we can push such calculations quite far. This is a game-changer.

Studies of quark/gluon scattering amplitudes in pQCD revealed field-theoretic structures that were not expected.

Leading order: Berends-Giele recursion

For leading order computations, many key ideas (spinor helicity methods, color-stripped amplitudes, Berends-Giele (BG) recursions) that we use to describe multi-parton final states at the LHC appeared in the late 1980s. Thanks to the significant increase in computing power and improved algorithms for phase-space generation, calculation of amplitudes and cross-sections for very high multiplicity processes is now possible.



σ [μb]	Number of jets						
jets	2	3	4	5	6	7	8
Comix	331.0(4)	22.72(6)	4.95(2)	1.232(4)	0.352(1)	0.1133(5)	0.0369(3)
ALPGEN	331.7(3)	22.49(7)	4.81(1)	1.176(9)	0.330(1)		
AMEGIC	331.0(4)	22.78(6)	4.98(1)	1.238(4)			

σ [μb]	Number of jets						
$b\bar{b}$ + jets	0	1	2	3	4	5	6
Comix	471.2(5)	8.83(2)	1.813(8)	0.459(2)	0.150(1)	0.0531(5)	0.0205(4)
ALPGEN	470.6(6)	8.83(1)	1.822(9)	0.459(2)	0.150(2)	0.053(1)	0.0215(8)
AMEGIC	470.3(4)	8.84(2)	1.817(6)				

σ [pb]	Number of jets						
$t\bar{t}$ + jets	0	1	2	3	4	5	6
Comix	754.8(8)	745(1)	518(1)	309.8(8)	170.4(7)	89.2(4)	44.4(4)
ALPGEN	755.4(8)	748(2)	518(2)	310.9(8)	170.9(5)	87.6(3)	45.1(8)
AMEGIC	754.4(3)	747(1)	520(1)				

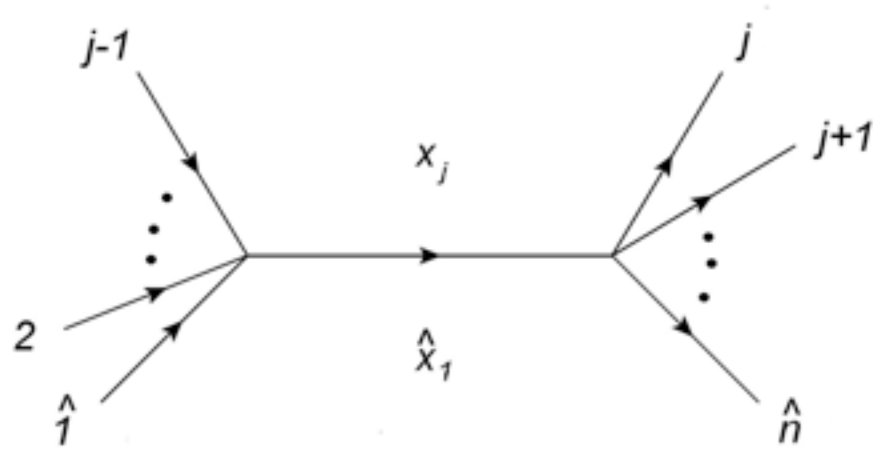
Tab. 7 Cross sections σ in the MC4LHC comparison [32] setup. In parentheses the statistical error is stated in units of the last digit of the cross section. Note that for AMEGIC++ and COMIX all subprocesses are considered, while ALPGEN is restricted to up to four quarks.

T. Gleisberg and S.Hoeche

BCFW recursion

Since leading order computational methods were very well established, it came as a big surprise that there is a way to perform them in a very different way.

Both, Berends-Giele and diagrammatic approaches to perturbative computations involve off-shell degrees of freedom. This seems to be a unavoidable consequence of the LSZ reduction formula. However, the BCFW procedure gets around it -- it allows on-shell amplitudes to be computed recursively from on-shell amplitudes of lower multiplicity.



$$\mathcal{M}(1,\ldots\hat{n}) = \mathcal{M}(1,..,j-1)\frac{i}{(p_1+p_2+..p_{j-1})^2}\mathcal{M}(j,\ldots\hat{n})$$

$$e_1^{+} = e_j^{-} = q/\sqrt{2}, \quad p_1\cdot q = p_j\cdot q = 0, \quad p_1 \to p_1+\mu q z, \quad p_j \to p_j - \mu q z$$

$$\oint_{|z|=\infty} \frac{\mathrm{d}z}{z}\mathcal{M}(z) = 0 \quad \Rightarrow \quad \mathcal{M}(0) = -i\sum_{l=1}^{\mathrm{l_{max}}}\sum_{\lambda=\pm}\frac{\mathcal{M}(\pi_1^l,\lambda)M(\pi_j^l,\lambda)}{P^2_{\pi_j^l}} \qquad P_{\pi_j^l} = \sum_{i\in\pi_j^l} p_i$$

Britto, Cachazo, Feng, Witten

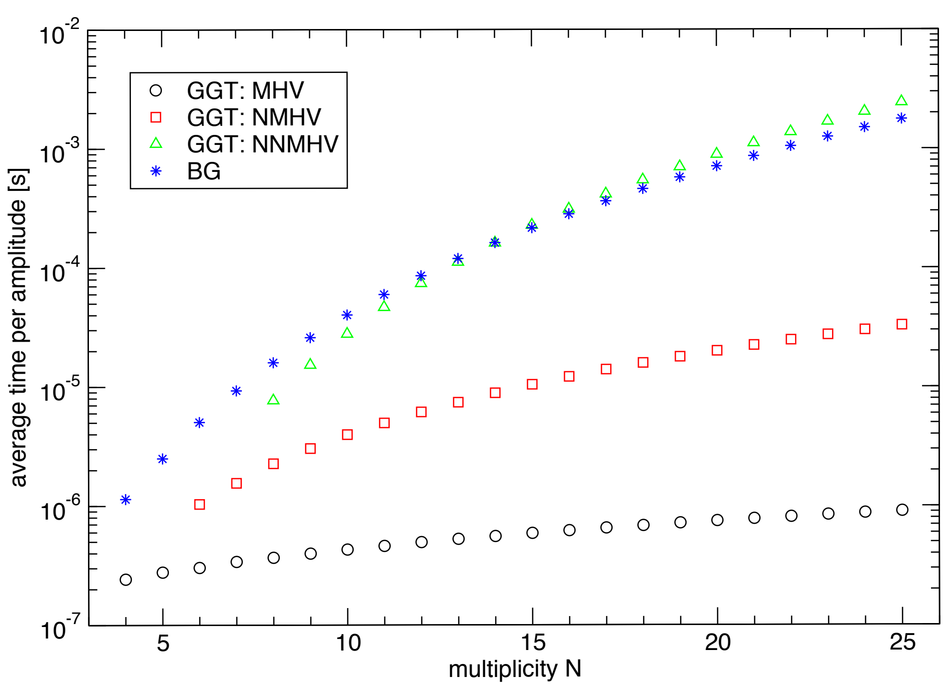
Progress with leading order computations

Explicit analytic solution for all tree QCD scattering amplitudes became available, as a byproduct of N=4 SYM computations. A comparison of analytic and BG results reveals that MHV and NMHV amplitudes are computed more efficiently with analytic methods; after that, BG recursion becomes more efficient.

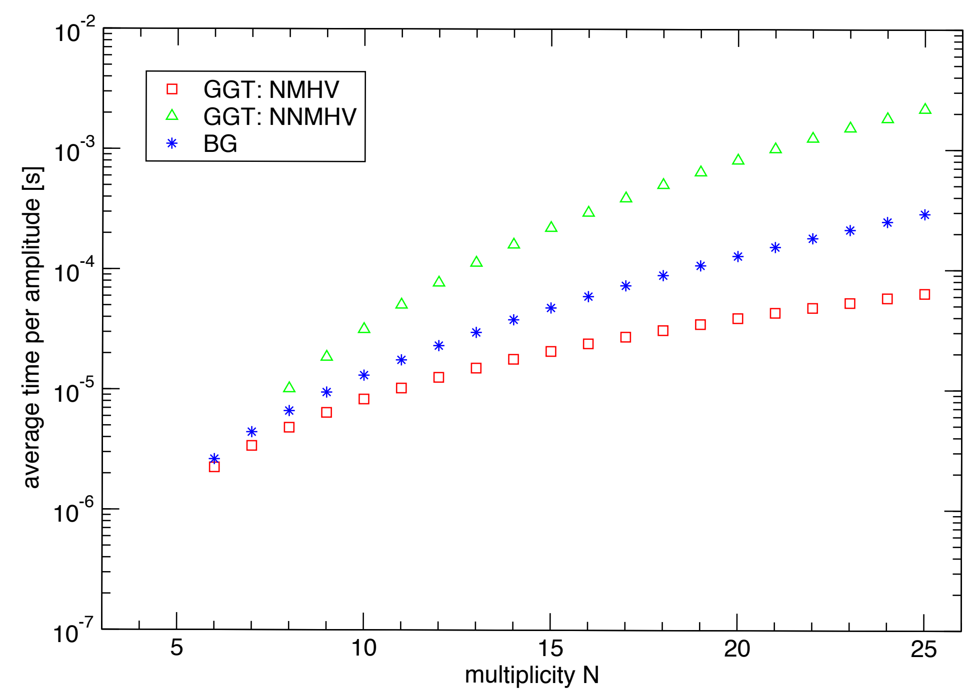
Dixon, Henn, Plefka, Schuster; Bourjaily

$$A_{jk}^{\mathrm{MHV}} = i\frac{\langle jk\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

N gluon amplitudes



(N-6) gluon 6 quark amplitudes



Badger, Biedermann, Hackl, Plefka, Schuster, Uwer

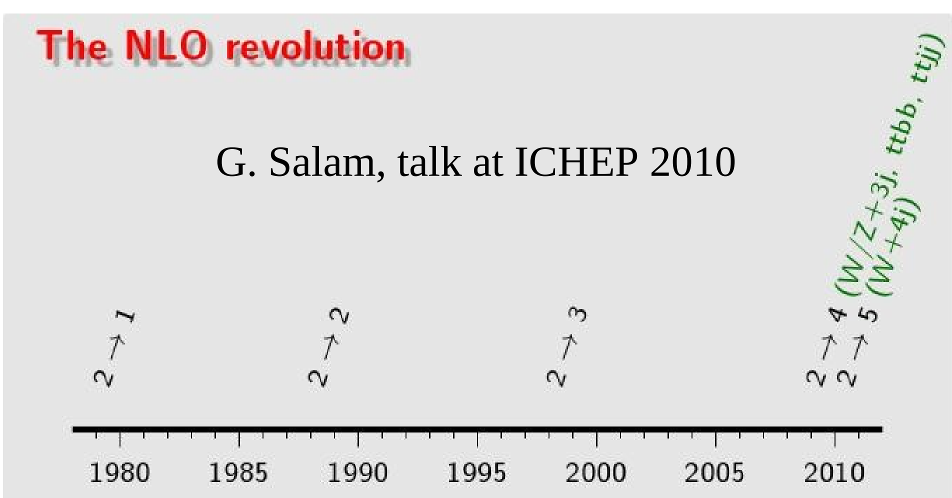
Progress with next-to-leading order

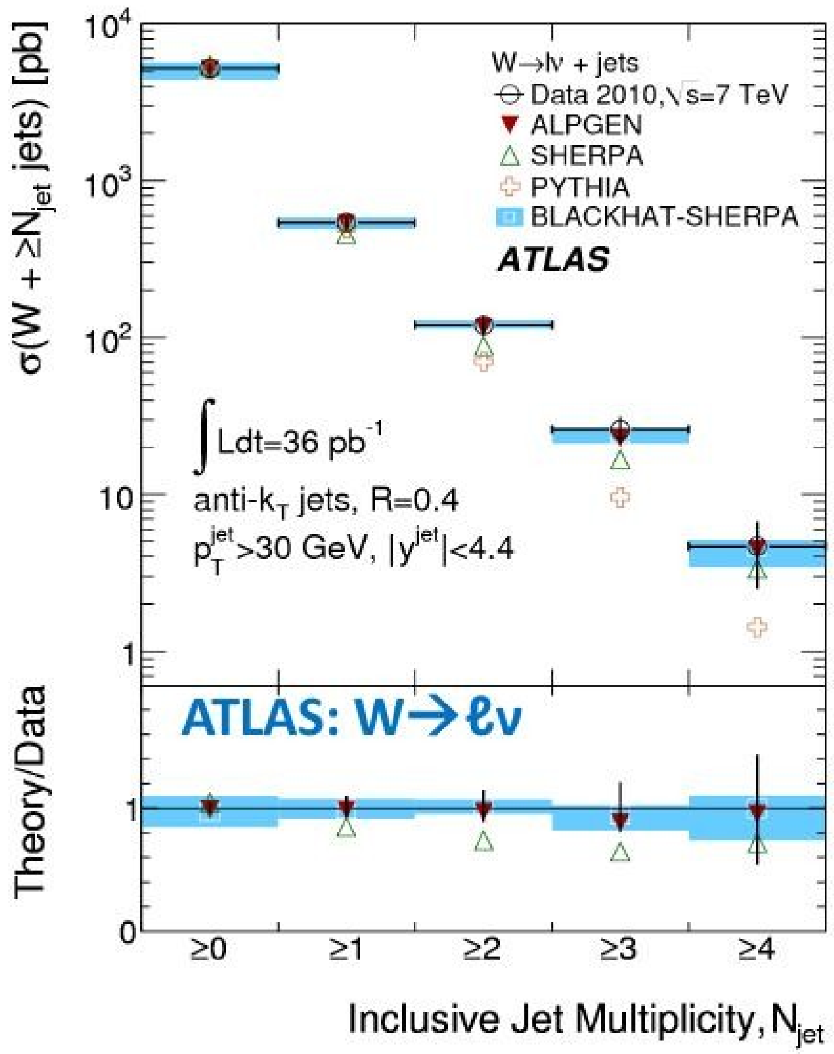
Progress with leading order computations did not translate into progress with next-to-leading order computations right away. Until recently, it took a decade to increase the final state multiplicity by one particle in a typical NLO computation.

In the past few years a remarkable change in pace occurred thanks to two developments:

1) better ways of dealing with Feynman diagrams;

2) radically new (on-shell) methods for one-loop computations.





W+5j can be added to this plot since recently

A change in the one-loop paradigm

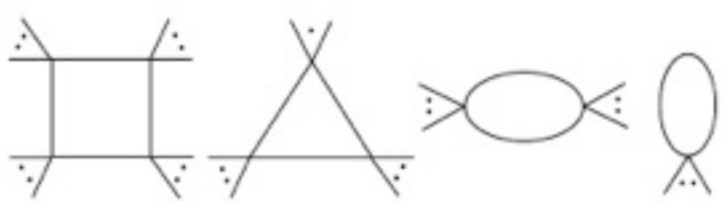
Unitarity (on-shell) techniques allow us to reconstruct one-loop scattering amplitudes directly from tree-amplitudes, by-passing Feynman diagrams.

Bern, Dixon, Kosower

Earlier, unitarity was considered to be a useful tool for low-multiplicity computations, but we now understand how to make use of it for high-multiplicity processes as well.

$$i\left(T_{ij} - T^{+}_{ij}\right) = \sum T_{in} T^{+}_{nj}$$

$$\mathcal{A}^{1-\mathrm{loop}} = \sum c_j I_j \qquad I_i =$$



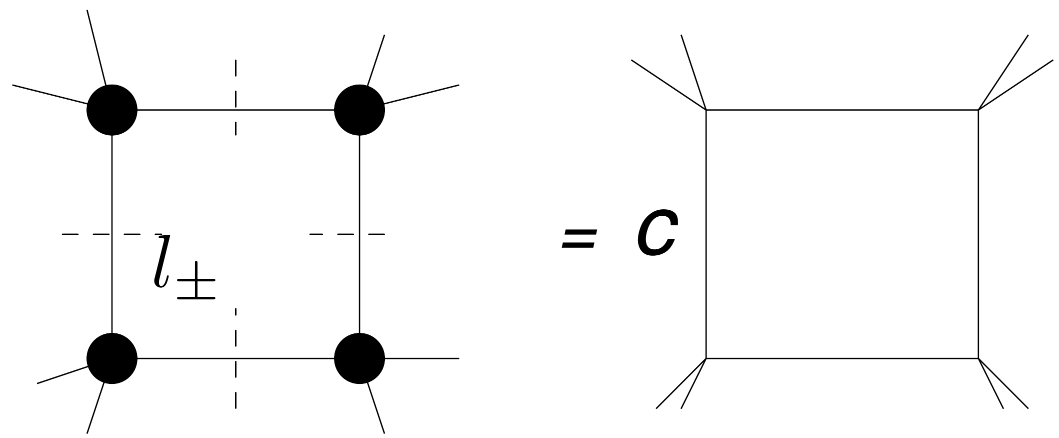
$$\mathrm{Im}\left(A^{1-\mathrm{loop}}\right) \propto \sum |\mathcal{A}^{\mathrm{tree}}|^2$$

$$\boxed{\sum c_j \mathrm{Im}(I_j) \propto \sum |A_{\mathrm{tree}}|^2}$$

The big boost to this technology came from a new way of tensor reduction for one-loop integrals discovered by Ossola, Pittau and Papadopulous (OPP) and from the observation of Ellis, Giele and Kunszt that generalized unitarity at one-loop can be derived from the OPP tensor reduction.

The box coefficient

The simplicity and elegance of generalized unitarity is particularly striking when computation of box reduction coefficients are considered.



Replace a set of four propagators by delta-functions $\frac{1}{l^2-m^2} \to \delta(l^2-m^2)$ to isolate a particular box integral. Four delta-functions restrict the integration completely.

$$c = \frac{1}{2}\sum_{l=l_\pm} A_1^{\mathrm{tree}} A_2^{\mathrm{tree}} A_3^{\mathrm{tree}} A_4^{\mathrm{tree}}$$

Britto, Cachazo, Feng

The reduction coefficient is expressed through tree on-shell scattering amplitudes that appear in the four corners of the one-loop diagram in the left hand side above. The result is general -- the number of external particles is irrelevant.

This simplicity does not quite hold up when extend this technique to triangles, bubbles, masses, rational parts...., i.e. everything that is required to compute full amplitudes. Nevertheless, to a large extent we can say that a tree-level S-matrix is sufficient to completely reconstruct the one-loop S-matrix.

One-loop calculations: no more wishes

These (and more traditional diagrammatic) developments of one-loop technology lead to a significant accomplishment -- NLO QCD predictions are now available for major hadron collider processes, making rich phenomenology possible.

1) multiple jets (up to 4);

2) a gauge boson and up to 5 (!) jets;

3)multiple gauge bosons in association with up to 2 jets (up to VV+2jets);

4) top quarks in association with jets (up to two) and electroweak gauge bosons (W,Z,photon);

5) the Higgs bosons with up to 2 jets.

Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \to VV$ jet	WW jet completed by Dittmaier/Kallweit/Uwer [27, 28]; Campbell/Ellis/Zanderighi [29]. ZZjet completed by Binoth/Gleisberg/Karg/Kauer/Sanguinetti [30]
2. $pp \to$ Higgs+2jets	NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi [31]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [32, 33] Interference QCD-EW in VBF channel [34, 35]
3. $pp \to VVV$	ZZZ completed by Lazopoulos/Melnikov/Petriello [36] and WWZ by Hankele/Zeppenfeld [37], see also Binoth/Ossola/Papadopoulos/Pittau [38] VBFNLO [39, 40] meanwhile also contains $WWW, ZZW, WW\gamma, ZZ\gamma, WZ\gamma, W\gamma\gamma, Z\gamma\gamma, \gamma\gamma\gamma$ $WZj, W\gamma j, \gamma jj$, [illegible]
4. $pp \to t\bar{t}b\bar{b}$	relevant for $t\bar{t}H$, computed by Bredenstein/Denner/Dittmaier/Pozzorini [41, 42] and Bevilacqua/Czakon/Papadopoulos/Pittau/Worek [43]
5. $pp \to V$+3jets	W+3jets calculated by the Blackhat/Sherpa [44] and Rocket [45] collaborations Z+3jets by Blackhat/Sherpa [46]
Calculations remaining from Les Houches 2005	
6. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$, computed by Bevilacqua/Czakon/Papadopoulos/Worek [47, 48]
7. $pp \to VV\,b\bar{b}$, 8. $pp \to VV$+2jets	Pozzorini et al.[25],Bevilacqua et al.[23] W^+W^++2jets [49], W^+W^-+2jets [50], VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [51, 52, 53]
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	Binoth et al. [54, 55]
NLO calculations added to list in 2009	
10. $pp \to V + 4$ jets	top pair production, various new physics signatures Blackhat/Sherpa: W+4jets [22], Z+4jets [20] see also HEJ [56] for $W + n$jets
11. $pp \to Wb\bar{b}j$	top, new physics signatures, Reina/Schutzmeier [11]
12. $pp \to t\bar{t}t\bar{t}$	various new physics signatures
also: $pp \to 4$ jets	Blackhat/Sherpa [19]

CLOSED

One-loop calculations: automation

	Process	μ	n_{lf}	Cross section (pb) LO	NLO
a.1	$pp \to t\bar{t}$	m_{top}	5	123.76 ± 0.05	162.08 ± 0.12
a.2	$pp \to tj$	m_{top}	5	34.78 ± 0.03	41.03 ± 0.07
a.3	$pp \to tjj$	m_{top}	5	11.851 ± 0.006	13.71 ± 0.02
a.4	$pp \to t\bar{b}j$	$m_{top}/4$	4	25.62 ± 0.01	30.96 ± 0.06
a.5	$pp \to t\bar{b}jj$	$m_{top}/4$	4	8.195 ± 0.002	8.91 ± 0.01
b.1	$pp \to (W^+ \to)e^+\nu_e$	m_W	5	5072.5 ± 2.9	6146.2 ± 9.8
b.2	$pp \to (W^+ \to)e^+\nu_e\, j$	m_W	5	828.4 ± 0.8	1065.3 ± 1.8
b.3	$pp \to (W^+ \to)e^+\nu_e\, jj$	m_W	5	298.8 ± 0.4	300.3 ± 0.6
b.4	$pp \to (\gamma^*/Z \to)e^+e^-$	m_Z	5	1007.0 ± 0.1	1170.0 ± 2.4
b.5	$pp \to (\gamma^*/Z \to)e^+e^-\, j$	m_Z	5	156.11 ± 0.03	203.0 ± 0.2
b.6	$pp \to (\gamma^*/Z \to)e^+e^-\, jj$	m_Z	5	54.24 ± 0.02	56.69 ± 0.07
c.1	$pp \to (W^+ \to)e^+\nu_e b\bar{b}$	$m_W + 2m_b$	4	11.557 ± 0.005	22.95 ± 0.07
c.2	$pp \to (W^+ \to)e^+\nu_e t\bar{t}$	$m_W + 2m_{top}$	5	0.009415 ± 0.000003	0.01159 ± 0.00001
c.3	$pp \to (\gamma^*/Z \to)e^+e^- b\bar{b}$	$m_Z + 2m_b$	4	9.459 ± 0.004	15.31 ± 0.03
c.4	$pp \to (\gamma^*/Z \to)e^+e^- t\bar{t}$	$m_Z + 2m_{top}$	5	0.0035131 ± 0.0000004	0.004876 ± 0.000002
c.5	$pp \to \gamma t\bar{t}$	$2m_{top}$	5	0.2906 ± 0.0001	0.4169 ± 0.0003
d.1	$pp \to W^+W^-$	$2m_W$	4	29.976 ± 0.004	43.92 ± 0.03
d.2	$pp \to W^+W^-\, j$	$2m_W$	4	11.613 ± 0.002	15.174 ± 0.008
d.3	$pp \to W^+W^+\, jj$	$2m_W$	4	0.07048 ± 0.00004	0.1377 ± 0.0005
e.1	$pp \to HW^+$	$m_W + m_H$	5	0.3428 ± 0.0003	0.4455 ± 0.0003
e.2	$pp \to HW^+\, j$	$m_W + m_H$	5	0.1223 ± 0.0001	0.1501 ± 0.0002
e.3	$pp \to HZ$	$m_Z + m_H$	5	0.2781 ± 0.0001	0.3659 ± 0.0002
e.4	$pp \to HZ\, j$	$m_Z + m_H$	5	0.0988 ± 0.0001	0.1237 ± 0.0001
e.5	$pp \to Ht\bar{t}$	$m_{top} + m_H$	5	0.08896 ± 0.00001	0.09869 ± 0.00003
e.6	$pp \to Hb\bar{b}$	$m_b + m_H$	4	0.16510 ± 0.00009	0.2099 ± 0.0006
e.7	$pp \to Hjj$	m_H	5	1.104 ± 0.002	1.036 ± 0.002

MadLoop, Hirshi et al.

Sherpa–OpenLoops process library for A

Status and available processes

- careful process-by-process validation (most processes r
- full set of NLO QCD diagrams, full colour
- off-shell leptonic W/Z decays: interferences, complex r
- on-shell top quarks with LO decays

W/Z	γ	jets	HQ pairs	single-top
V+3j	γ+3j	3(4)j	t$\bar{\text{t}}$+1j	tb+1j
VV+2j	$\gamma\gamma$+1(2)j		t$\bar{\text{t}}V$+0(1)j	t+1(2)j
gg $\to VV$+1j	$V\gamma$+2j		b$\bar{\text{b}}V$+0(1)j	tW+0(1)j
VVV+0(1)j				

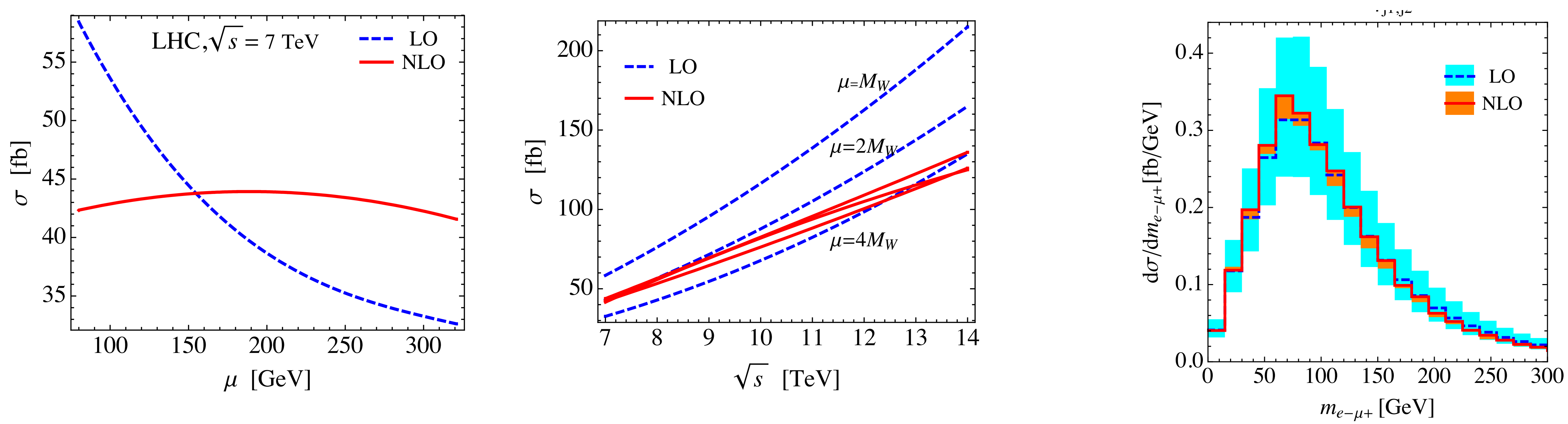
lower jet multiplicities implicitly understoo

From a talk by S. Pozzorini

The other consequence of this progress is that high efficiency of the NLO QCD computational methods for the first time makes it possible to have a real shot at Madgraph-style automation of NLO QCD calculations.

Learning from the NLO results

NLO results for cross sections have reduced renormalization and factorization scale dependence and, for the first time, provide reliable predictions for normalization of cross sections. For this reason, NLO computations will be indispensable when measurements of couplings and cross sections will start in earnest.



$pp \to W^+W^- + 2j$

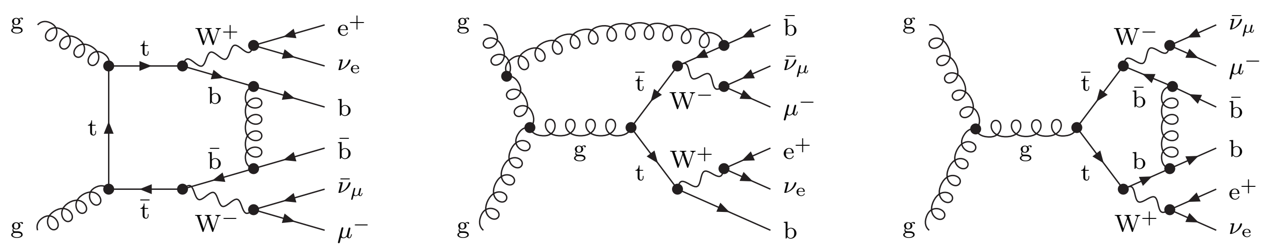
T. Melia, R. Ronstch et al.

But this is not all: indeed, since NLO QCD computations take us quite a bit closer to reality, we should be able to learn more from them. In addition to better phenomenology, we should be able to assess quality of various approximations and short-cuts that we might want to use to arrive at reliable results in more complex cases.

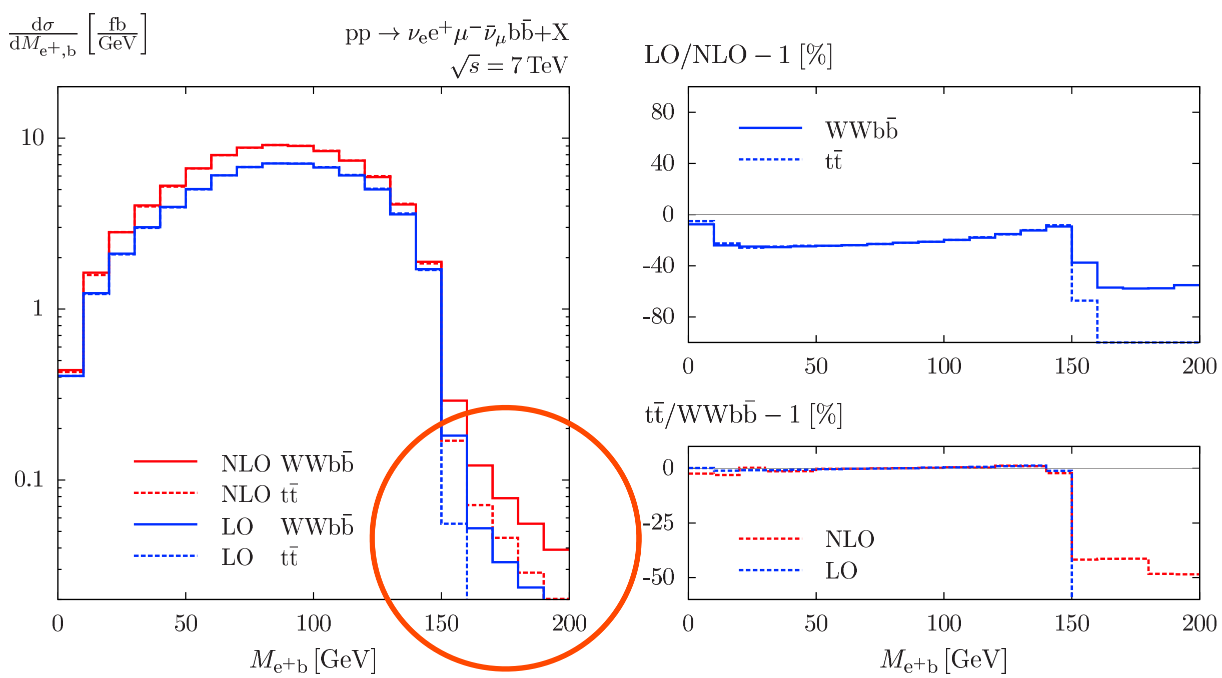
Off-shell effects in top quark pair production

How well does the narrow width approximation work for the LHC physics?

We can answer this question by comparing top quark pair production computed in the narrow width approximation with complete computation for WWbb final state.



Denner, Dittmaier, Kallweit, Pozzorini



Collider	$\sqrt{s}$ [TeV]	approx.	$\sigma_{t\bar{t}}$ [fb]	$\sigma_{WWb\bar{b}}$ [fb]	$\sigma_{t\bar{t}}/\sigma_{WWb\bar{b}} - 1$	Ref. [25]
Tevatron	1.96	LO	$44.691(8)^{+19.81}_{-12.58}$	$44.310(3)^{+19.68}_{-12.49}$	$+0.861(19)\%$	$+0.8\%$
		NLO	$42.16(3)^{+0.00}_{-2.91}$	$41.75(5)^{+0.00}_{-2.63}$	$+0.98(14)\%$	$+0.9\%$
LHC	7	LO	$659.5(1)^{+261.8}_{-173.1}$	$662.35(4)^{+263.4}_{-174.1}$	$-0.431(16)\%$	-0.4%
		NLO	$837(2)^{+42}_{-87}$	$840(2)^{+41}_{-87}$	$-0.41(31)\%$	-0.2%
LHC	14	LO	$3306.3(1)^{+1086.8}_{-763.6}$	$3334.6(2)^{+1098.5}_{-771.2}$	$-0.849(7)\%$	$---$
		NLO	$4253(3)^{+282}_{-404}$	$4286(7)^{+283}_{-407}$	$-0.77(19)\%$	$---$

Denner, Dittmaier, Kallweit, Pozzorini, Schulze

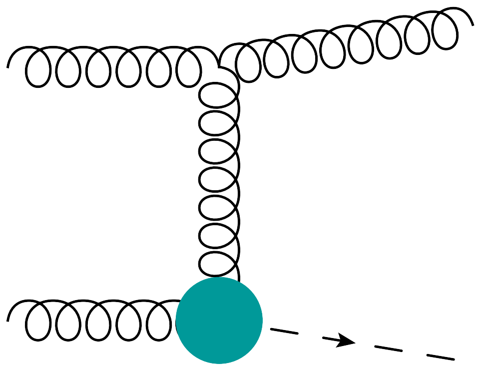
Standard top-like selection cuts are applied to WWbb final state for cross-section calculation.

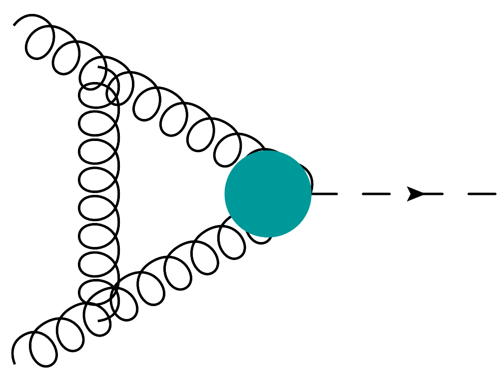
Significant effects beyond the clear kinematic edge of b-lepton invariant mass at LO in the narrow width approximation. Perhaps relevant for the recent ``cleanest" top quark mass measurement performed by the CMS from the end point?

Kinematic distributions agree very well when on-shell kinematics is allowed but may show larger deviations when this is not the case.

Higgs production in association with jets

Kinematic constraints on observable final states may introduce instabilities into perturbative expansion. The reason can be an``incomplete" cancellation of infra-red sensitivity of virtual and real corrections or sensitivity of an observable to soft/collinear emissions.





$$\sigma_{0j}^{R} \approx \sigma_B \frac{C_A \alpha_s}{\pi} \left(\frac{1}{\epsilon^2} - 2 \ln^2 \frac{p_\perp}{m_H} \right)$$

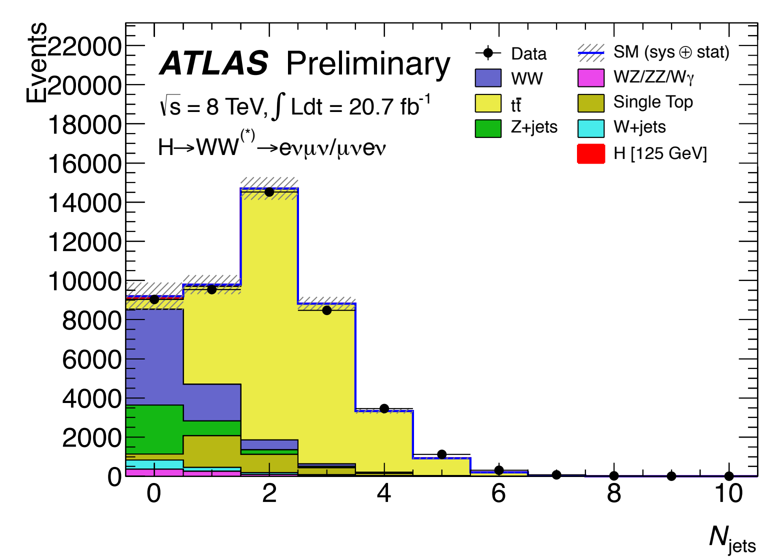
$$\sigma_{0j}^{V} \approx -\frac{C_A \alpha_s}{\pi \epsilon^2} \sigma_B$$

$$\sigma_{0j} \approx \sigma_B \left(1 - \frac{2 C_A \alpha_s}{\pi} \ln^2 \frac{p_\perp}{m_H} \right)$$

$$\sigma_{1j} \approx \sigma_B \frac{2 C_A \alpha_s}{\pi} \ln^2 \frac{p_\perp}{m_H}$$

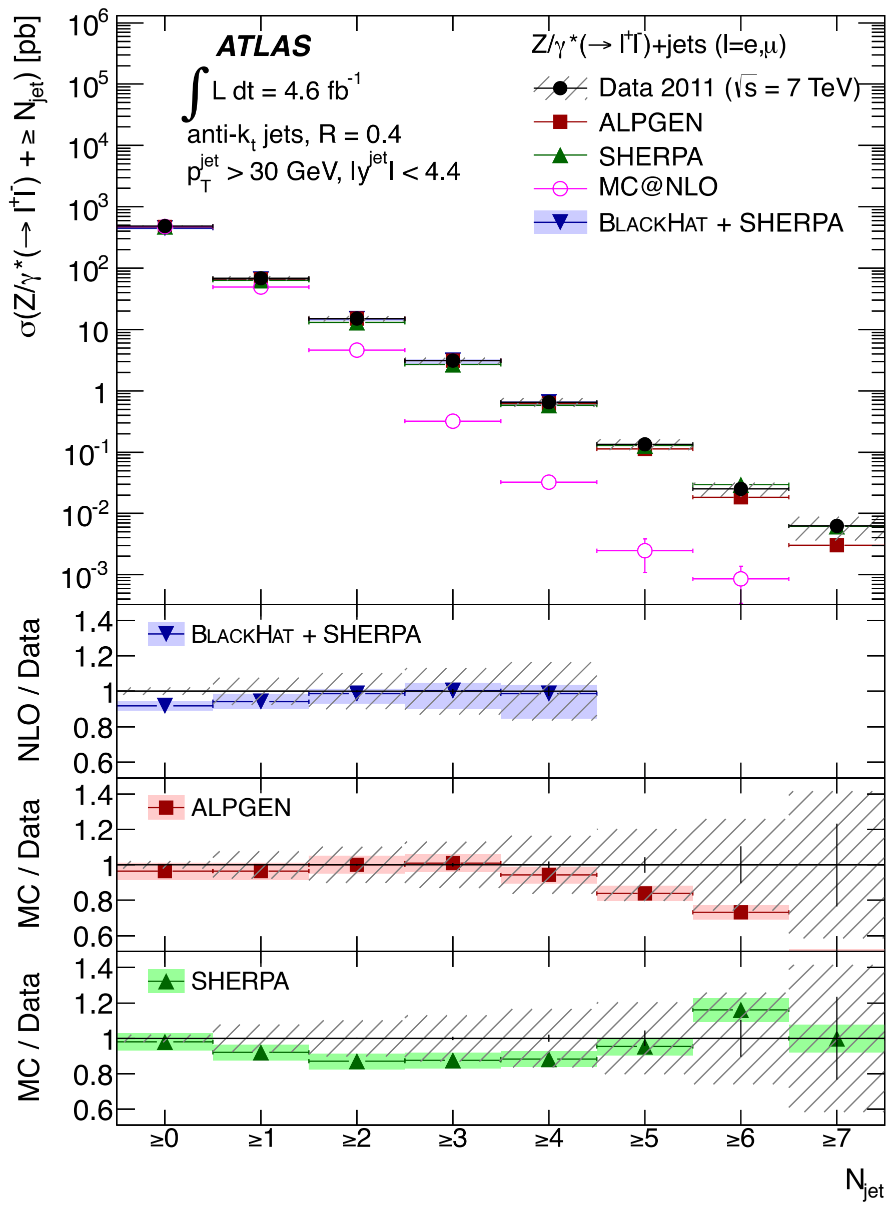
The sum of the jet vetoed cross-section and the 1-jet cross-section is the inclusive cross-section the is not affected by the transverse momentum cuts

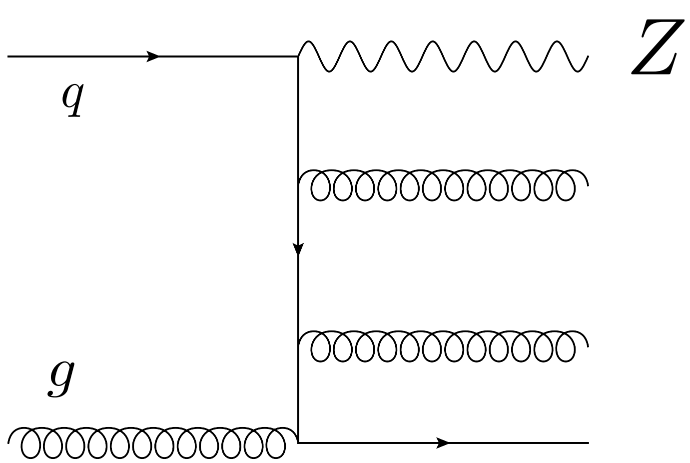
Numerically, for 30 GeV transverse momentum vero, the corrections may be significant but they are hardly overwhelming and, probably, can be cured by going to sufficiently high orders in fixed-order perturbation theory or performing resummations. Nevertheless, it is interesting to see if we have experimental indications that processes with jet-vetoes show worse perturbative behavior than the inclusive ones.



Jet vetos and large logarithms ?

Jet cross sections and jet-vetoed cross sections are two sides of the same coin. Jet cross sections for objects with electroweak-scale masses and 30 GeV jet transverse momentum cut have been measured at the LHC. There is no indication that perturbation theory breaks down.

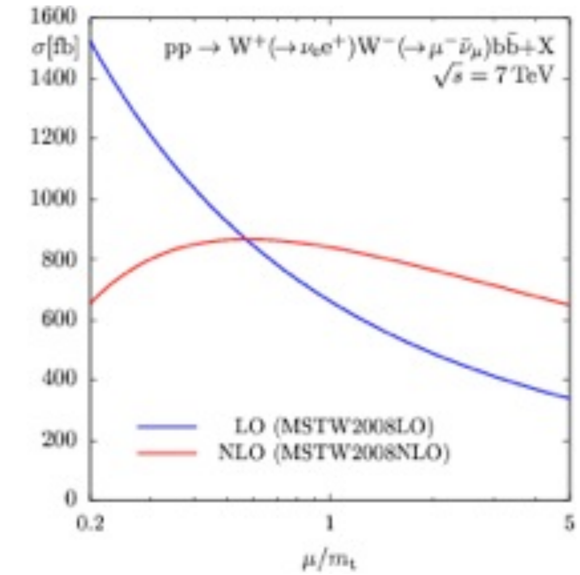




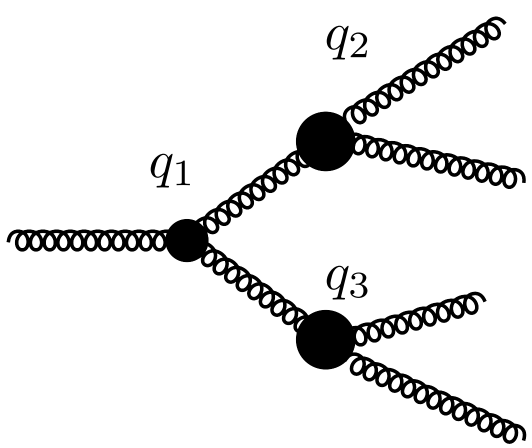
The important difference between Higgs and Z production is that color charges of colliding partons are smaller in Z-production than in the Higgs production -- radiative effects in Higgs production should be worse. But this difference is (somewhat) ameliorated if we go to Z+1j etc. Yet, even in Z+1 jet, perturbation theory works well.

Choosing renormalization and factorization scales

Since NLO computations are more scale-independent than LO computations,   we can ask the question -- what is the right scale choice at LO ?



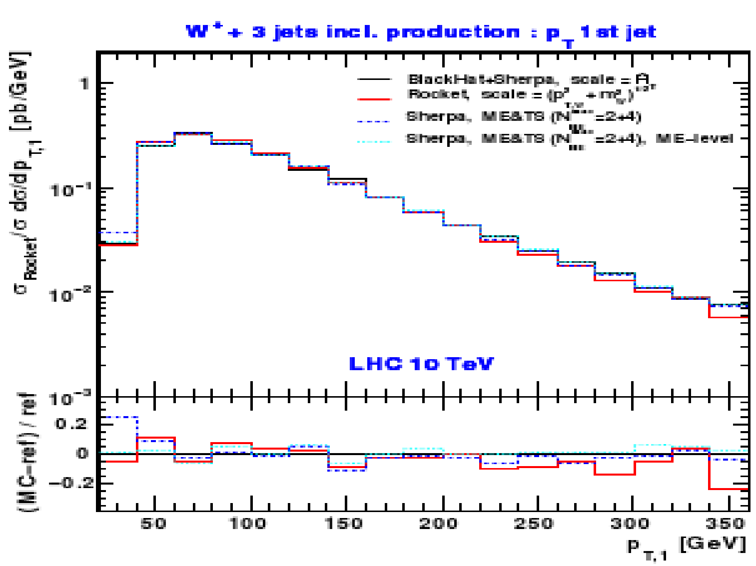
Of course, no universal answer exists since the coupling constants ``run''; the ``right scale'' is determined by an event location in a multi-particle phase-space.



$$\mathrm{Prob}(a \to b) \sim \alpha_s(p_\perp)$$

$$|\mathcal{M}|^2 \sim \prod_{i=1}^{N} \alpha_s(q_i) \prod_{ij} \frac{\Delta(q_i)}{\Delta(q_j)}$$

CKKW/MLM procedures have a build-in prescription to fix dynamical scales. They often provide shapes of kinematic distributions consistent with NLO QCD predictions



S. Hoche, J. Huston, D. Maitre, J. Winter and G. Zanderighi

It is possible to extend the scale-setting prescription of CKKW to NLO by choosing the geometric mean of nodal scales to compute the virtual corrections (MINLO)

Hamilton, Nason, Zanderighi

Can MINLO be a poor-man solution to exact NNLO rates and shapes? It will be interesting to see that.

Improving on next-to-leading order computations

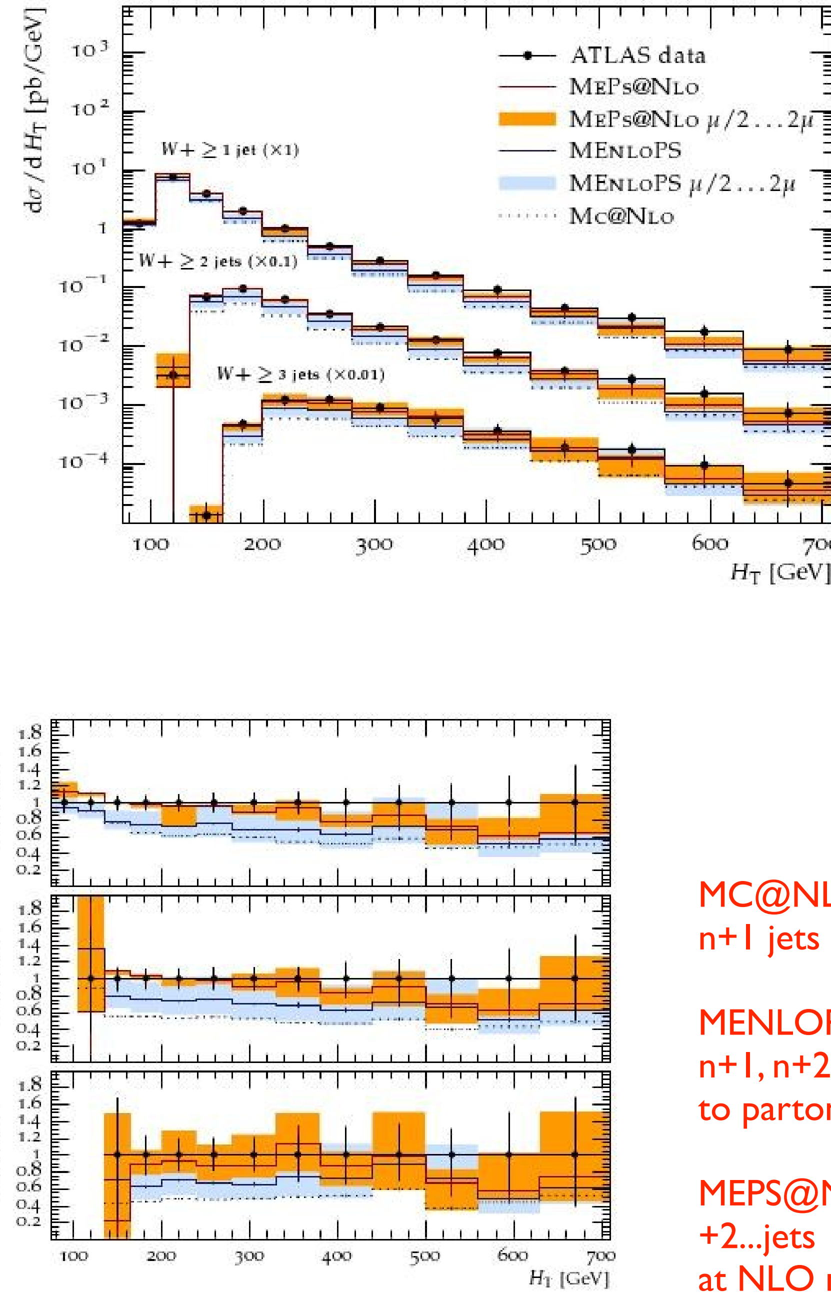
NLO computations have known shortcomings: 1) they fail close to kinematic boundaries; 2) they show residual dependence on renormalization scales; 3) they describe a very limited combination of parton multiplicities at a time.

We can cure these problems by

1) combining NLO computations with parton showers (MC@NLO, POWHEG, SHERPA) -- an ever increasing number of processes is being implemented in these programs;

2) understanding how to merge NLO QCD predictions for processes with different multiplicities (MEPS@NLO, GENEVA);

3) developing NNLO technology for multi-particle processes.



MC@NLO: n-jets NLO, n+1 jets at LO, rest -- PS

MENLOPS: n-jets NLO, n+1, n+2...jets LO matched to parton shower

MEPS@NLO: n, n+1, n +2...jets at NLO merged and matched to parton shower

Hoche, Krauss, Schonherr, Siegert

Next-to-next-to-leading order computations

While next-to-leading order computations for the LHC proved to be a tough problem, the ``in principle'' solution was available for a long time. In the 1970s Passarino and Veltman described a reduction procedure for tensor integrals and t'Hooft and Veltman explained how to compute one-loop scalar integrals. A general procedure to reduce arbitrary NLO computation to calculation of infra-red and collinear-finite quantities was formulated by Catani and Seymour and by Frixione, Kunszt and Signer in mid-1990s.

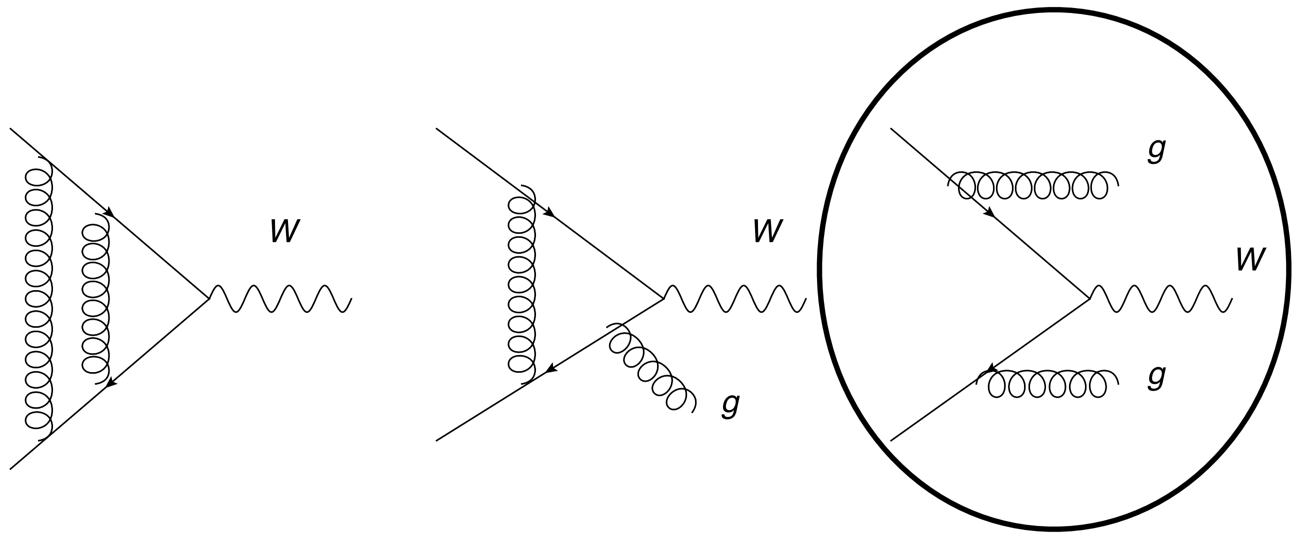
On the contrary, technique for next-to-next-to-leading order computations was a total unknown for a long time. To be sure, a large number of multi-loop effects were computed for LEP physics, but all of them referred to inclusive decays of the Z-bosons.

The only ``20th century'' NNLO QCD hadron collider computation was done by van Neerven and collaborators in the 1990; the next one had to wait for 12 years.

It is therefore quite remarkable that in the past year we have witnessed a tremendous progress towards fully general computational scheme that should allow us to deal with generic NNLO QCD calculations. The ``NNLO revolution'' is taking over the NLO one.

I would like to illustrate these developments by showing you the key steps in reaching the NNLO QCD frontier in hadron collider physics.

Next-to-next-to-leading order computations



$$I_n^{(i,j)} = \int_0^{\pi} d\theta \int_0^{\pi} d\phi \, \frac{(\sin\theta)^{n-3}(\sin\phi)^{n-4}}{(a+b\cos\theta)^i (A+B\cos\theta+C\sin\theta\cos\phi)^j},$$

A very difficult analytic computation that was hardly automated (1990!)

A COMPLETE CALCULATION OF THE ORDER α_s^2 CORRECTION TO THE DRELL–YAN *K*-FACTOR

R. HAMBERG and W.L. van NEERVEN*

Instituut–Lorentz, University of Leiden, P.O.B. 9506, 2300 RA Leiden, The Netherlands

T. MATSUURA**

II. Institut für Theoretische Physik, Universität Hamburg, D-2000 Hamburg 50, Germany

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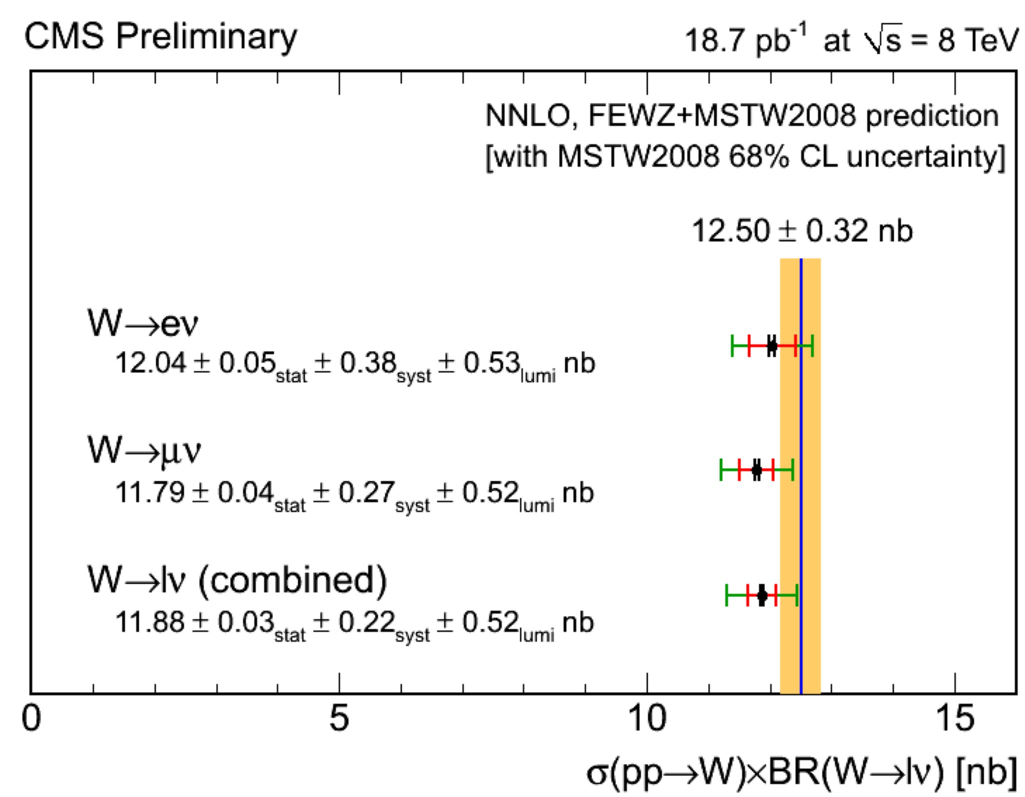
In this paper we present the complete calculation of the order α_s^2 correction in the $\overline{\text{MS}}$ scheme to the Drell–Yan *K*-factor. All channels represented by the $q\bar{q}$, qg, gg and qq subprocesses have been included now. One of our conclusions is that the $O(\alpha_s^2)$ part of the *K*-factor is dominated by the $q\bar{q}$ as well as the qg reaction. The latter leads to a negative contribution over the whole energy range under investigation (0.5 TeV $< \sqrt{S} <$ 50 TeV). It even overwhelms the positive $q\bar{q}$ contribution at large collider energies characteristic for LHC and SSC. It turns out that the order α_s^2 corrected *K*-factor is quite insensitive to variations of the factorization scale *M* over the region 10 GeV $< M <$ 1000 GeV. We also compare our results with the data obtained by UA1, UA2 and CDF.

Results slightly corrected by Kilgore and Harlander in 2002



The NNLO computation for DY production is also used to constraint parton distribution functions.

Next-to-next-to-leading order computations

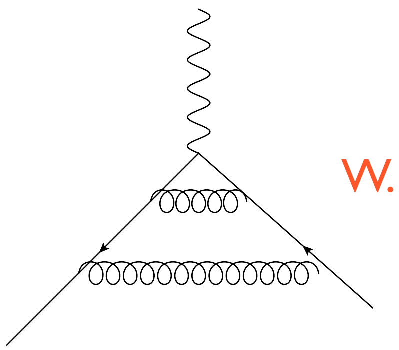
The NNLO Drell-Yan computation required two-loop quark form-factor. It was computed by van Neerven earlier using dispersion relations. But -- it was unclear how to extend this technique to compute two-loop integrals relevant for 2 - 2 scattering.

In 1999, Smirnov and Tausk showed how to compute two-loop planar and non-planar box diagrams.

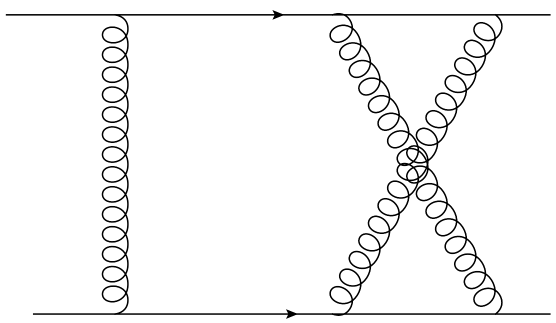
At around the same time, Laporta came up with the suggestion on how to ``automate'' solutions of integration-by-parts identities discovered by Chetyrkin and Tkachov.

These developments lead to very rapid progress with calculations of a large number of 2-2 scattering amplitudes for massless particles $gg \to gg,\ q\bar{q} \to gg\ qg \to qg,\ qg \to q\gamma^*, gg \to Hg,$ etc.

Anastasiou, Glover, Gehrmann, Gehrmann-de Ridder, Oleari, Remiddi, Bern, Dixon etc.



W. van Neerven



B. Tausk

By 2001 it appeared that NNLO computations for 2-2 processes were within reach but we had to wait for another decade to see this happening....

Next-to-next-to-leading order computations

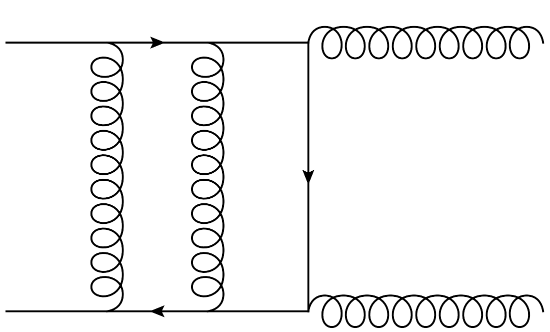
The reason it took so long to understand how to do the NNLO computations was the problem with constructing the "subtraction terms" for real emission processes.

Subtraction terms are needed because -- while infra-red divergencies do cancel in the sum of the three contributions shown below -- these three contributions live in different phase-spaces and can not be computed in the same manner.

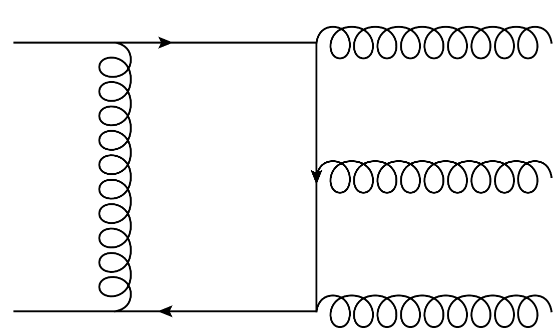
To deal with this issue, we introduce subtraction terms that make double-real and real-virtual contributions finite and integrable over their particular phase-spaces even when constrained by the measurement function (jet algorithms, experimental selection cuts, etc.).

$$\int \mathrm{d}\Phi_{n+2} R = \int \mathrm{d}\Phi_{n+2} \left(R - \tilde{R} \right) + \int \mathrm{d}\Phi_n \tilde{R}_I$$

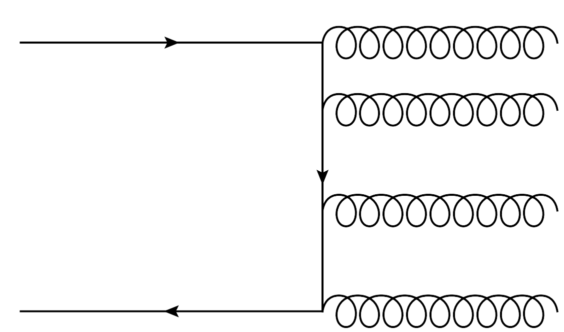
Attempts to understand how to do this at NNLO proved to be very challenging.



Double virtual



Real-virtual

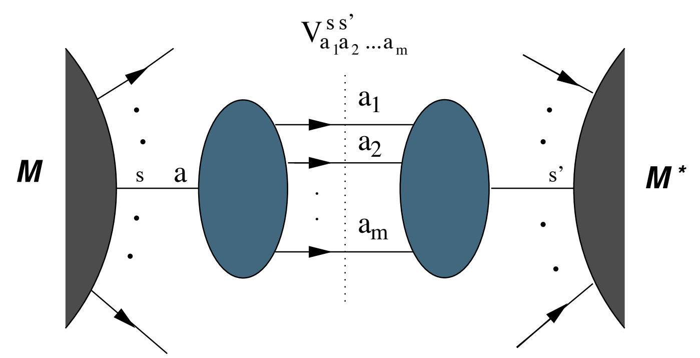


Real-real

Singular limits of scattering amplitudes

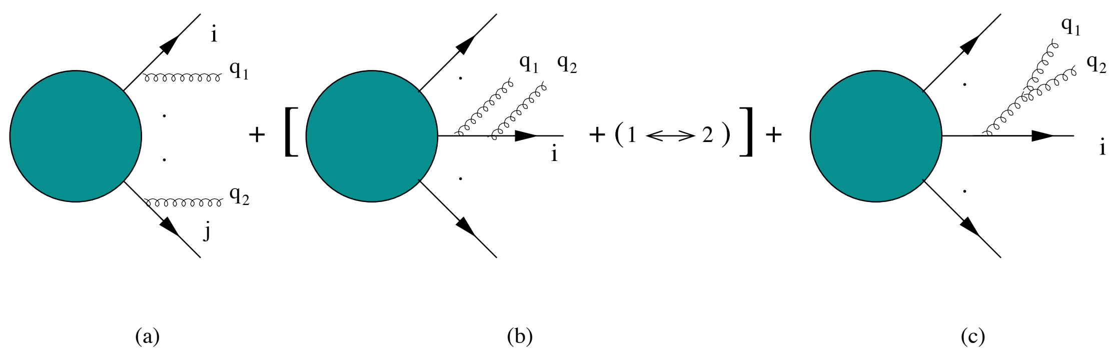
Subtraction terms must be relevant in singular parts of the phase-space, where some partons become soft or collinear to other particles. For this reason, one would hope that subtraction terms can be obtained from singular limits of the amplitudes.

It is interesting to point out that all singular limits of scattering amplitudes relevant for NNLO calculations were worked out explicitly by the year 2000. The issue that prevented us from using these singular limits to construct subtraction terms was the problem of overlapping divergences.

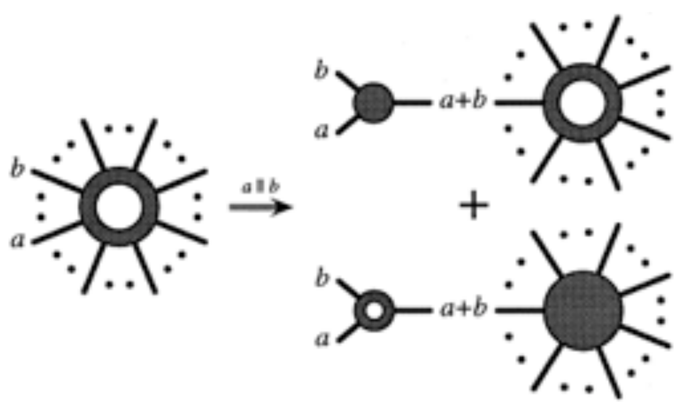


Collinear factorization (Catani, Grazzini)

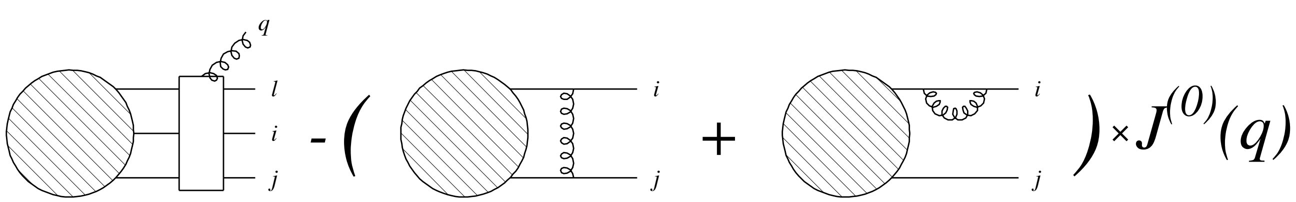
$$\mathcal{M}(\{n\}+j+j) \approx g_s^2 F_{\text{sing}}(\{n\},i,j)\mathcal{M}(\{n\})$$



Soft factorization (Catani, Grazzini)



Collinear factorization at one-loop (Kosower, Uwer)



Soft factorization at one-loop (Catani, Grazzini)

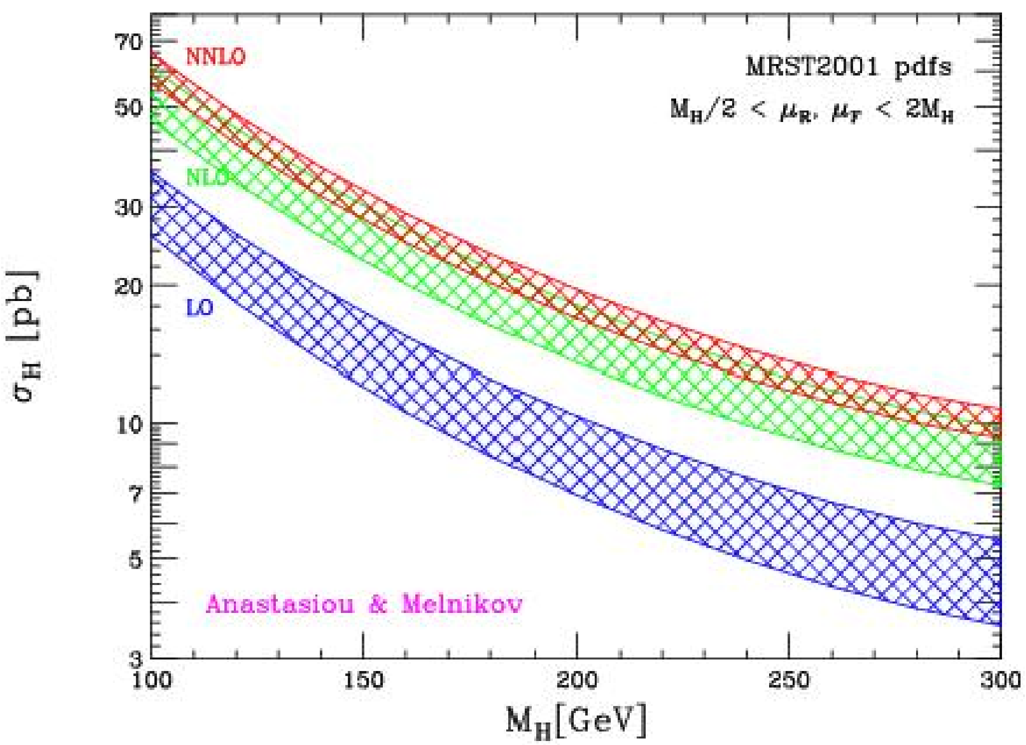
Related work on singular limits by Campbell, Glover, Berends, Giele, Bern, Del Duca, Kilgore, Schmidt

Next-to-next-to-leading order computations

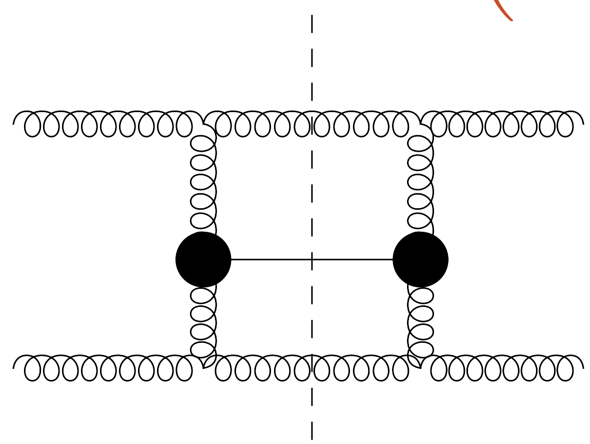
In the meantime, the progress was coming from a different direction. Computation of NNLO QCD corrections to Higgs boson production in gluon fusion is technically similar to Drell-Yan, but it is more involved (gluon self-couplings are more nasty). Particularly bad is the computation of real emission ggH phase-space integrals.

To deal with this problem, Anastasiou and myself suggested to use the ``reversed unitarity": write on-shell condition as a difference of two propagators, apply integration-by-parts identities to forward scattering amplitude, express the result in terms of a few ``phase-space" master-integrals....

$$\mathcal{L} = \frac{\alpha_s}{6\pi} \mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] \log\left(1+\frac{H}{v}\right),$$



Also, Harlander and Kilgore; van Neerven, Rabindran, Smith



$$\delta(p^2-m^2) \to \frac{1}{2\pi i}\left(\frac{1}{p^2+m^2-i0} - \frac{1}{p^2-m^2+i0}\right)$$

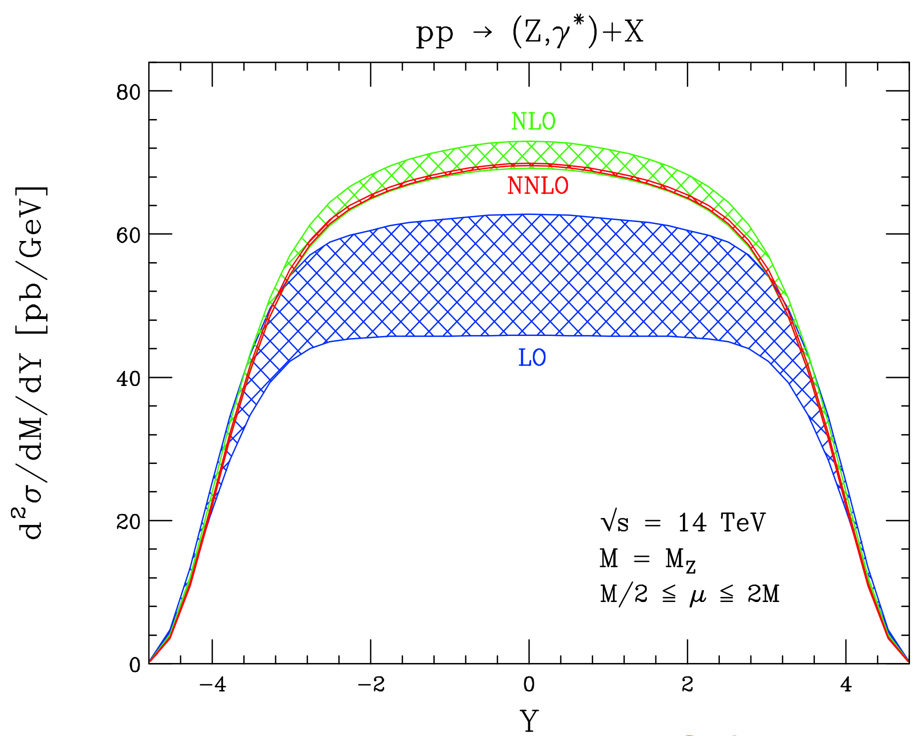
According to a recent study by C. Anastasiou and collaborators, it is possible to make use of the ``reversed unitarity" idea for the N^3LO ggH computation

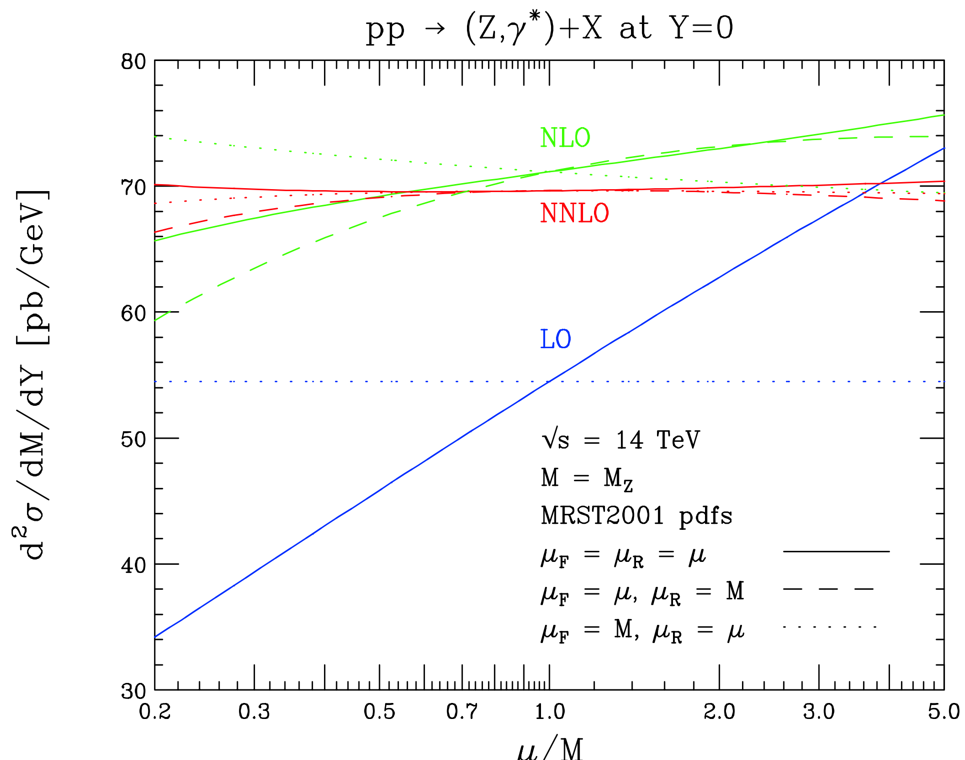
Next-to-next-to-leading order computations

Next-to-next-to-leading order computations for total cross-sections are important, but at hadron colliders they are largely unphysical, because of finite detector acceptances. To continue pushing for higher precision, we need to control higher-order effects in a phase-space, restricted by an arbitrary selection criteria.

First solution to the problem was peculiar -- choose a distribution, use the ``reversed unitarity" but now write kinematic constraints as differences of unconventional propagators, treat everything as a forward-scattering amplitude, and use integration by parts to reduce the problem to the evaluation of master integrals.

$$u = \frac{x_1}{x_2} e^{-2Y} \qquad \delta\left(\frac{p_V \cdot p_1}{p_V \cdot p_2} - u\right) \to \frac{1}{2\pi i}\left(\frac{p_V \cdot p_2}{p_V \cdot (p_1 - u p_2) - i0} - c.c.\right)$$





C. Anastasiou, L.Dixon, K.M., F. Petriello

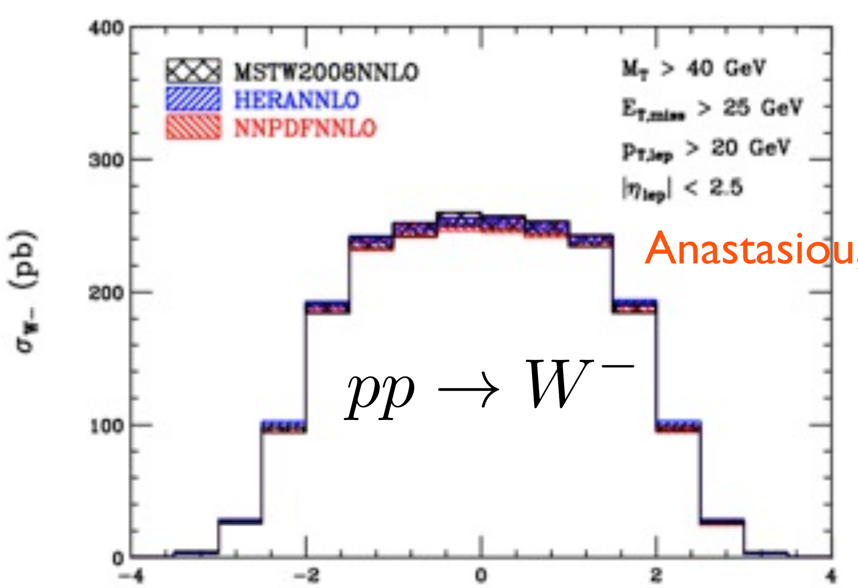
Next-to-next-to-leading order computations

Yet, even with Z and W rapidity distributions computed through NNLO, the question of fully-differential NNLO computation remained. We want to construct a flexible computational scheme that allows us to extract all the singularities yet not integrate over final state particles.. For hadron colliders, this problem was first solved in two different way, but both had serious limitations.

Sector decomposition -- as originally formulated -- requires global parametrization of a multi-particle phase-space. On the other hand, it can only work efficiently if parametrization of singular ``corners" of the phase-space is simple. The two conditions are not compatible for complex processes where the singularity structure is non-trivial.

The pt-subtraction works only for colorless final states...

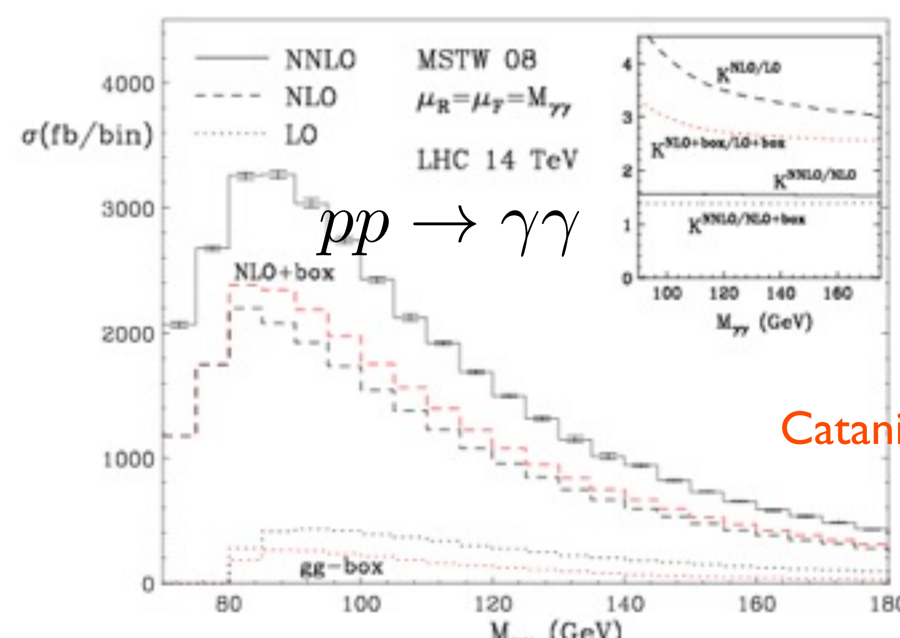
sector decomposition (FEHiP, FEWZ)



Anastasiou, Melnikov, Petriello

FEWZ2.1 Ryan, Li, Petriello, Quackenbush

pt-subtraction (HNNLO, DYNNLO)



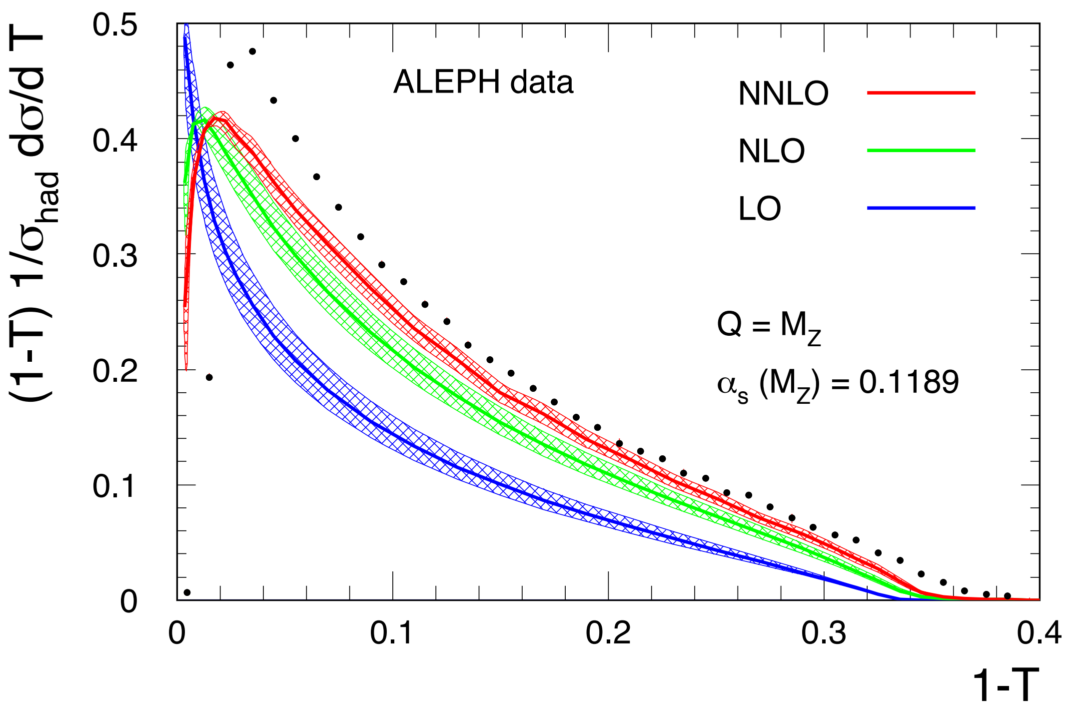
Catani, Grazzini

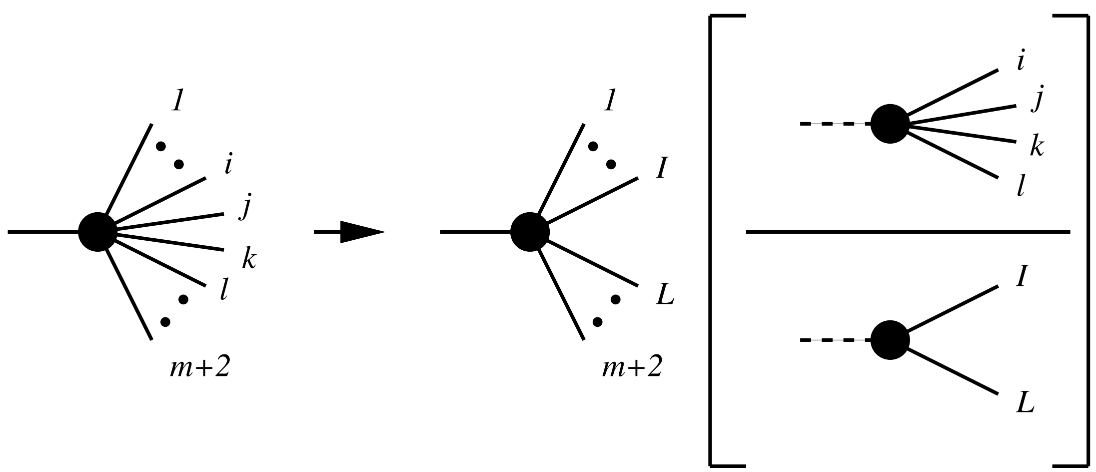
S. Catani, D. Florian, G.Ferrera and M.Grazzini

Next-to-next-to-leading order computations

In parallel to this, new tools were developed for NNLO QCD computations at lepton colliders. Heinrich, Gehrmann-de Ridder , Gehrmann and Glover suggested an antenna subtraction method to compute the 3-jet production rate and many event shapes in electron-positron annihilation.

Antenna subtraction method is the first (and the only) NNLO method that is based on the identification of subtraction terms, their analytic integration over unresolved phase-space and analytic cancellation of infra-red and collinear singularities.





Next-to-next-to-leading order computations

As you see, work on identifying suitable framework to deal with real emission contributions to NNLO QCD corrections for a generic hadron collider process was in the making for almost a decade and there is a feeling that we finally have it.

One of the clear indications of this progress is that we now know how to use the singular limits of scattering amplitudes (that have been known for more than a decade) to construct the necessary subtraction terms.

This understanding is independent of the number of external particles in the process and, therefore, we can hope that we are on to the universal method !

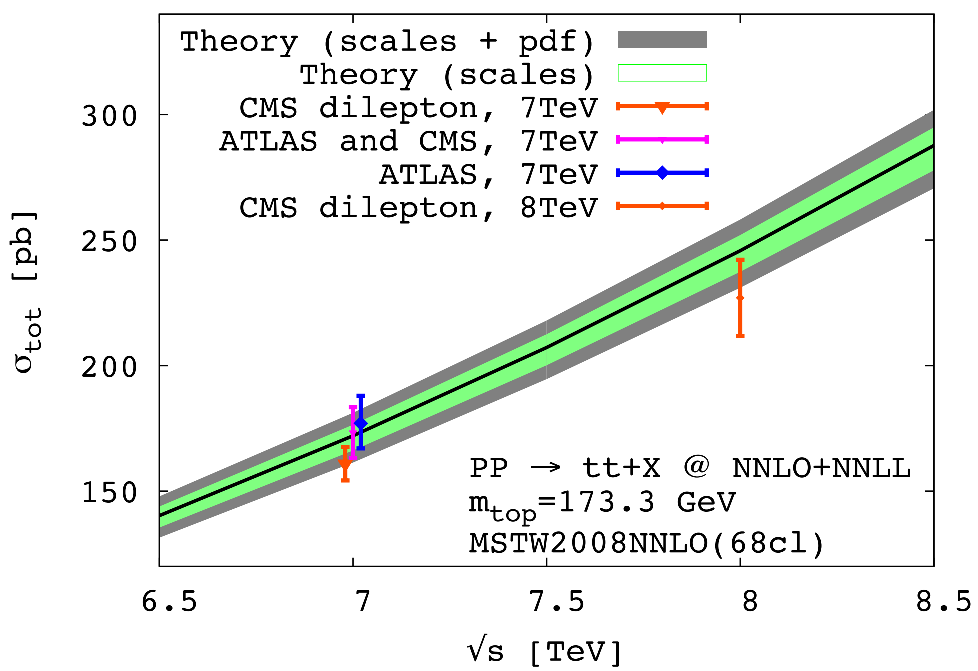
At NNLO QCD we have two working solutions

- antenna subtraction by Gehrmann-de-Ridder, Gehrmann, Glover ;

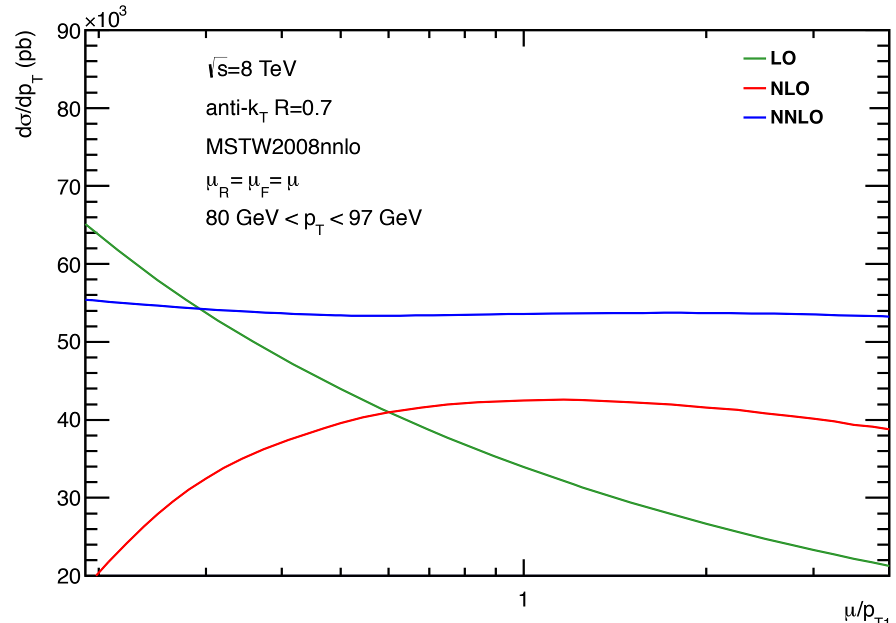
-sector decomposition/FKS by Czakon [also, Boughezal, Petriello, K.M.].

First NNLO results for 2 ⟶ 2 processes

First NNLO QCD results for top quark pair production (complete), dijet production (gluons only , large-N) and H+jet production (gluons only) in hadron collisions appeared recently. While some of these results are still incomplete it is clear that they represent a breakthrough in perturbative QCD that will, eventually, provide important phenomenological insights.

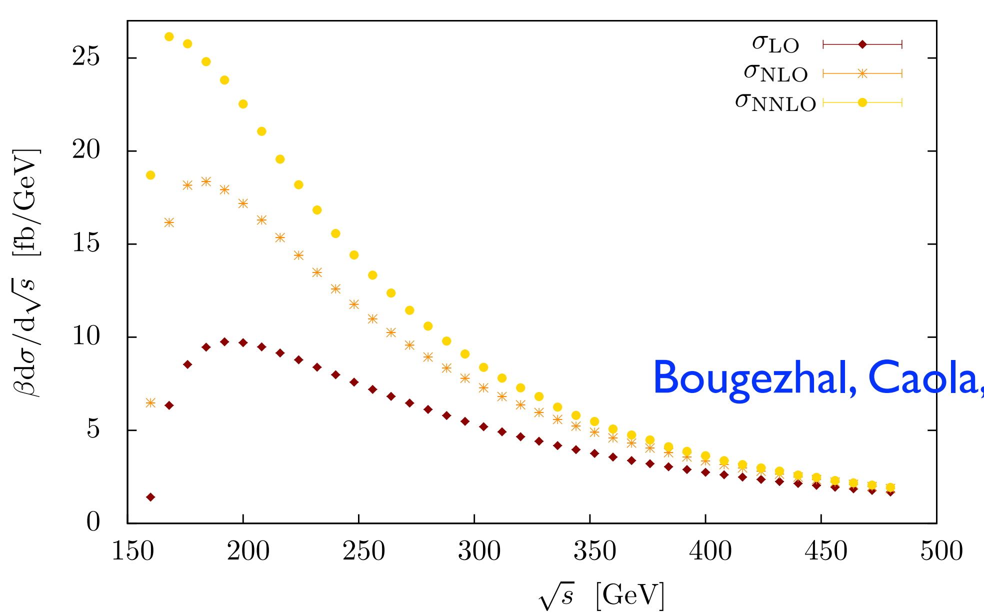


Czakon, Mitov, Fiedler



Gehrmann-der Ridder, Gehrmann, Glover, Pires

Interesting phenomenology started to appear for top quark pair production already (gluon pdfs, stealthy stops etc.)



Bougezhal, Caola, K.M., Petriello, Schulze

The next big thing

The next big problem with NNLO computations are virtual corrections to multi-jet processes. Both, reduction to and evaluation of master integrals, are difficult and will require new ideas.

Even with traditional methods for loop computations, we see significant differences between NLO and NNLO. Indeed, at NLO Passarino-Veltman reduction is algebraic but at NNLO no algebraic reduction is possible and integration-by-parts technique is needed for a complete solutions.

We should expect similar issues with extensions of OPP to two-loops -- parametric integration of spurious terms is the key at NLO but ``real" integration will likely be needed at NNLO.

Indeed, recent attempts to extend the on-shell methods and the OPP procedure to compute two-loop diagrams for multi-jet production showed significant increase in the number of irreducible scalar products. Planar N=4 SYM remains a spectacular exception but it is not so clear how to benefit from it for N=0.

The next big thing

An interesting (and largely forgotten) alternative is provided by numerical methods.

Weinzierl et al. showed that one can formulate the integration procedure directly in momentum space, both at one-loop and beyond.

One-loop integrals are made finite by subtraction terms and contour deformation.

At one-loop, the method was used to obtain large-N cross-sections for the production of up to seven (!) jets in electron-positron annihilation.

y_{cut}	$\frac{N_c^4}{8} A_{5,\mathrm{lc}}$	$\frac{N_c^5}{16} B_{5,\mathrm{lc}}$
0.002	$(5.0529 \pm 0.0004) \cdot 10^3$	$(4.275 \pm 0.006) \cdot 10^5$
0.001	$(1.3291 \pm 0.0001) \cdot 10^4$	$(1.050 \pm 0.026) \cdot 10^6$
0.0006	$(2.4764 \pm 0.0002) \cdot 10^4$	$(1.84 \pm 0.15) \cdot 10^6$

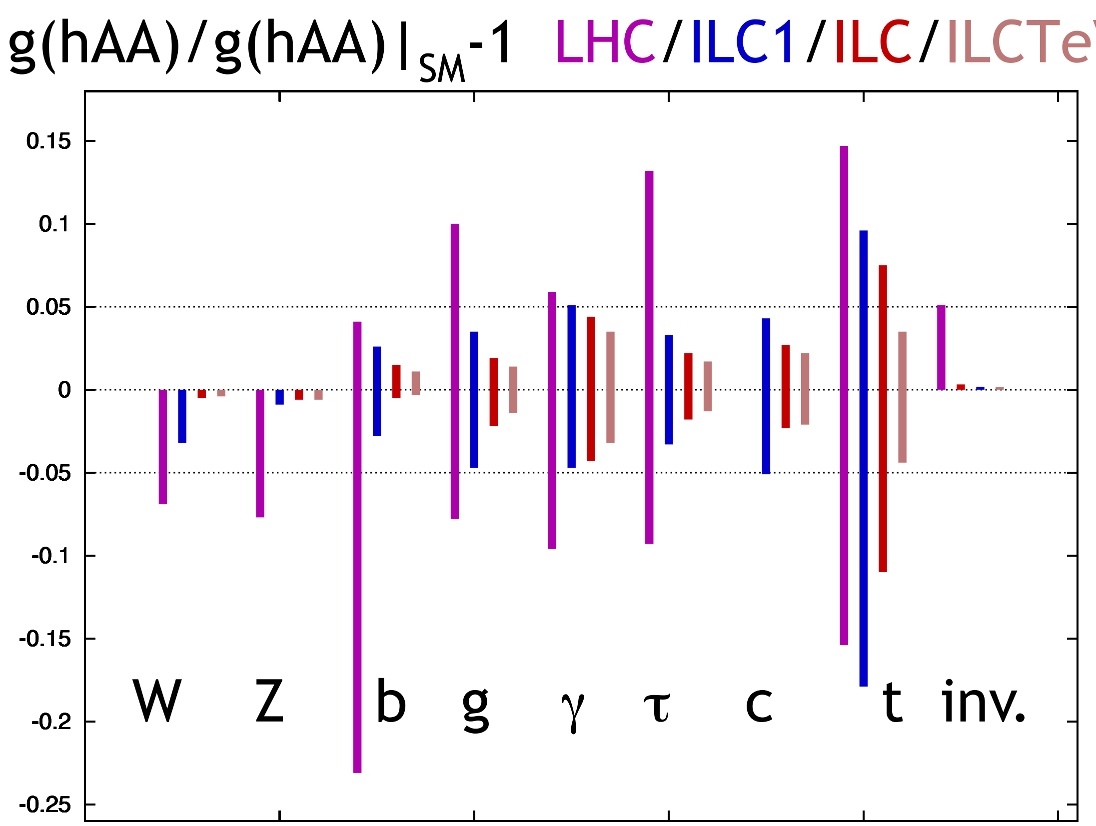
y_{cut}	$\frac{N_c^5}{16} A_{6,\mathrm{lc}}$	$\frac{N_c^6}{32} B_{6,\mathrm{lc}}$
0.001	$(1.1470 \pm 0.0002) \cdot 10^5$	$(1.46 \pm 0.04) \cdot 10^7$
0.0006	$(2.874 \pm 0.002) \cdot 10^5$	$(3.88 \pm 0.18) \cdot 10^7$

y_{cut}	$\frac{N_c^6}{32} A_{7,\mathrm{lc}}$	$\frac{N_c^7}{64} B_{7,\mathrm{lc}}$
0.0006	$(2.49 \pm 0.08) \cdot 10^6$	$(5.4 \pm 0.3) \cdot 10^8$

$$R_n = \left(\frac{\alpha_s}{2\pi}\right)^{n-2} A_n + \left(\frac{\alpha_s}{2\pi}\right)^{n-1} B_n$$

Becker, Goetz, Reschle, Schwan, Weinzierl

The Higgs boson signal: precision target



Observable	Expected Error (experiment $\oplus$ theory)
LHC at 14 TeV with 300 fb^{-1}	
$\sigma(gg) \cdot BR(\gamma\gamma)$	$0.06 \oplus 0.13$
$\sigma(WW) \cdot BR(\gamma\gamma)$	$0.15 \oplus 0.10$
$\sigma(gg) \cdot BR(ZZ)$	$0.08 \oplus 0.08$
$\sigma(gg) \cdot BR(WW)$	$0.09 \oplus 0.11$
$\sigma(WW) \cdot BR(WW)$	$0.27 \oplus 0.10$
$\sigma(gg) \cdot BR(\tau^+\tau^-)$	$0.11 \oplus 0.13$
$\sigma(WW) \cdot BR(\tau^+\tau^-)$	$0.15 \oplus 0.10$
$\sigma(Wh) \cdot BR(b\bar{b})$	$0.25 \oplus 0.20$
$\sigma(Wh) \cdot BR(\gamma\gamma)$	$0.24 \oplus 0.10$
$\sigma(Zh) \cdot BR(b\bar{b})$	$0.25 \oplus 0.20$
$\sigma(Zh) \cdot BR(\gamma\gamma)$	$0.24 \oplus 0.10$
$\sigma(t\bar{t}h) \cdot BR(b\bar{b})$	$0.25 \oplus 0.20$
$\sigma(t\bar{t}h) \cdot BR(\gamma\gamma)$	$0.42 \oplus 0.10$
$\sigma(WW) \cdot BR(\text{invisible})$	$0.2 \oplus 0.24$

The precision with which the Higgs couplings will be measured at the LHC are estimated to be limited by comparable theoretical and experimental uncertainties. It is hard (for me) to argue about experimental errors and how they will evolve. However, it is perfectly clear that progress in theoretical understanding of hadron collider data in the past decade was truly remarkable and, if it continues at a similar pace, high-precision predictions for very sophisticated final states will be available in ten years from now. This will be beneficial for Higgs couplings extraction at the LHC.

Conclusions

- Perturbative QCD provides good description of the wealth of data on hard scattering processes collected at the Tevatron and the LHC.
- Important for this success is the recent progress with NLO QCD technology as it allows us to make realistic and accurate description of complex final states.
- Same progress drives theoretical developments in matching fixed order computations to parton showers and merging theoretical predictions for various jet multiplicities.
- A working technology to perform complex NNLO QCD computations finally appeared. We now have time to consolidate the NNLO technology and move on to NNLO phenomenology.

All the developments in perturbative QCD that so many people worked on diligently through the past decade are becoming key for detailed understanding the properties of the Higgs boson and for searching for clues about BSM physics in the very Standard Model-like data .