From Low to High Energies

$$\lambda$$

Latsis Symposium
Nature at the Energy Frontier
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Outline

Dipole moments of leptons

electric, magnetic, and transitions: comparison

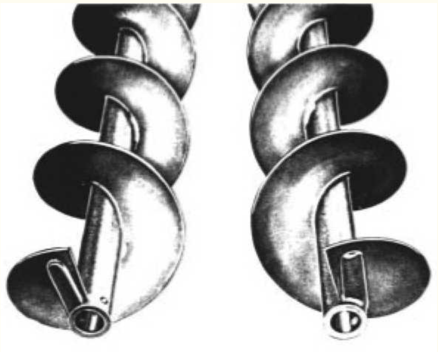
Orbital muon-electron conversion

Positronium hyperfine splitting

Anatomy of the electron

Two spinor fields $\psi_L,\ \phi_R$ transform differently under Lorentz boosts

$$R, L = \frac{1 \pm \gamma_5}{2}$$



How can Lorentz scalars be constructed?

$$\bar{\psi}_L \phi_R = \psi_L^\dagger \gamma^0 \phi_R$$

Another possibility, important for neutrinos:
See Alexei Smirnov's talk tomorrow.

$$\psi_L^T \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \psi_L$$

Constructing the electron mass term

A scalar structure we have found can be coupled to the Higgs field,

$$\lambda\bar{\psi}_L\phi_R + \text{H.c.} = |\lambda|\left(e^{i\delta}\bar{\psi}_L\phi_R + e^{-i\delta}\bar{\phi}_R\psi_L\right)$$

In order to make the mass real,
absorb the phase into one of the fields,

$$e^{i\delta}\phi_R \equiv \phi'_R$$

$$\Psi = \phi'_R + \psi_L$$

$$m \equiv |\lambda|$$

This fixes the relative phase of L, R components.

$$m\bar{\Psi}\Psi$$

Electron's interactions with other fields

Component fields L, R can be used to understand interaction terms,

Vector,

$$\bar{\Psi}\gamma^\mu\Psi = \bar{\psi}_L\gamma^\mu\psi_L + \bar{\phi}'{}_R\gamma^\mu\phi'_R$$

Tensor,

$$\bar{\Psi}\sigma^{\mu\nu}\Psi = \bar{\psi}_L\sigma^{\mu\nu}\phi'_R + \bar{\phi}'{}_R\sigma^{\mu\nu}\psi_L$$

Pseudotensor,

$$\bar{\Psi}\sigma^{\mu\nu}\gamma^5\Psi = \bar{\psi}_L\sigma^{\mu\nu}\phi'_R - \bar{\phi}'{}_R\sigma^{\mu\nu}\psi_L$$

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Pseudotensor, $$\bar{\Psi}\sigma^{\mu\nu}\gamma^5\Psi = \bar{\psi}_L\sigma^{\mu\nu}\phi'_R - \bar{\phi}'_R\sigma^{\mu\nu}\psi_L$$

How does this apply to electromagnetic moments?

MDM $a_e\frac{e}{2m}\sigma^{\mu\nu}F_{\mu\nu}$ EDM $id_e\sigma^{\mu\nu}\gamma^5F_{\mu\nu}$

A unified notation: $$\left(a_e\frac{e}{2m} + id_e\right)\bar{\psi}_L\sigma^{\mu\nu}\phi'_R + \mathrm{H.c.}$$

New Physics reach of dipole moments

First, consider the electron.

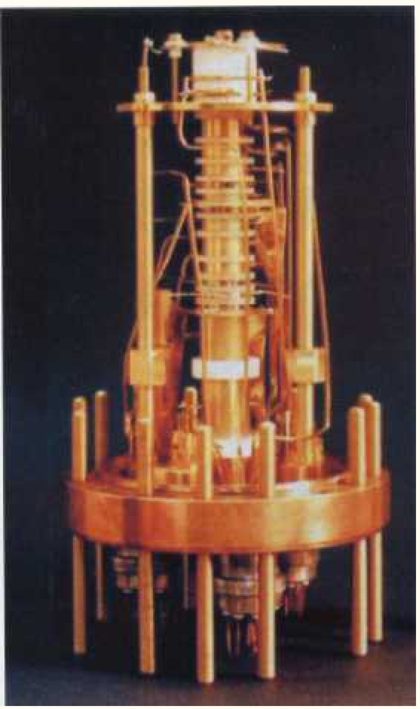
$$a_e = \frac{g_e - 2}{2}$$

is measured to the astounding 0.25 ppb and provides the fine structure constant with the same precision,

Phys. Rev. Lett. 100, 120801 (2008)

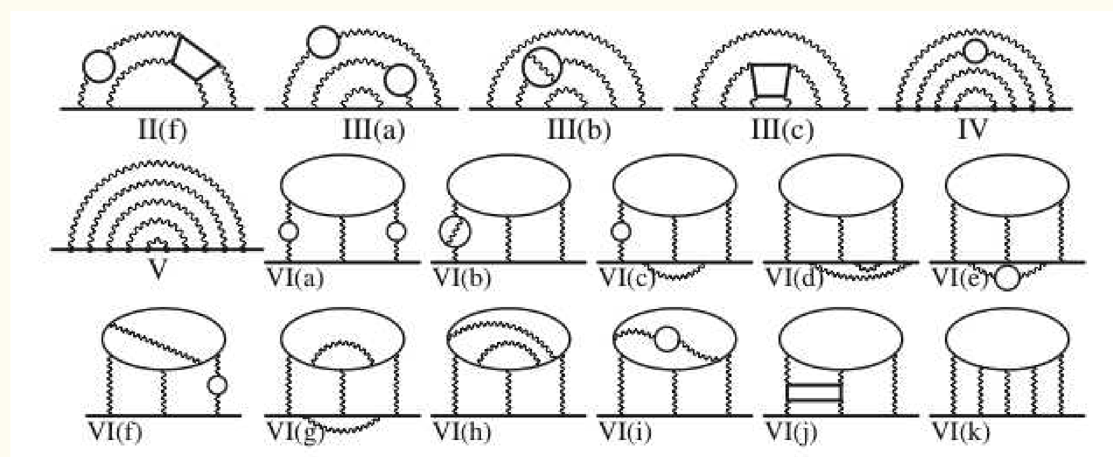
$$\alpha^{-1}(a_e) = 137.035\,999\,1736\,(331)\,(86)$$

Phys. Rev. Lett. 109, 111807 (2012)



Experimental error dominates (for now)

Numerical errors from 4- and 5-loop diagrams



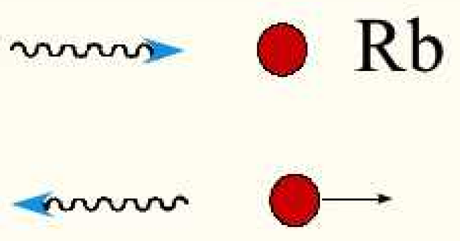
How to use this great result to search NP?

The second best determination of alpha:
from atomic spectroscopy

$$R_\infty = \frac{m_e c \alpha^2}{2h}$$

known to 6 ppt

and rubidium recoil



gives m/h

$$\alpha^{-1}(\mathrm{Rb10}) = 137.035\,999\,049\,(90) \quad [0.66\mathrm{ppb}]$$

PRL 106, 080801 (2011)

Together with the five-loop theory, this lets us make the comparison,

$$a_e^{\mathrm{th/Rb}} - a_e^{\mathrm{exp}} \simeq 10^{-12}$$

consistent with zero
at 1.3 sigma.

New Physics reach: comparison with EDM

What to expect for the electron EDM, given this agreement th/exp in the MDM?

Remember the unified notation, $\left(a_e \frac{e}{2m} + i d_e\right) \bar{\psi}_L \sigma^{\mu\nu} \phi'_R$

With the New Physics constrained by $a_e^{\mathrm{NP}} \lesssim 10^{-12}$

and if there are no further suppressions we can expect

$$d_e \sim \frac{e}{2m_e} a_e^{\mathrm{NP}} \sim 2 \cdot 10^{-23}\, \mathrm{e} \cdot \mathrm{cm}$$

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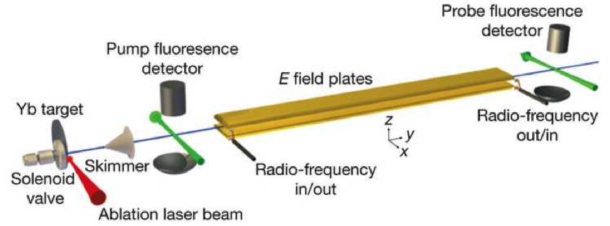
$$d_e \sim \frac{e}{2m_e} a_e^{\rm NP} \sim 2 \cdot 10^{-23}\, {\rm e \cdot cm}$$

The direct search finds $|d_e| < 10^{-27}\, e \cdot {\rm cm}$

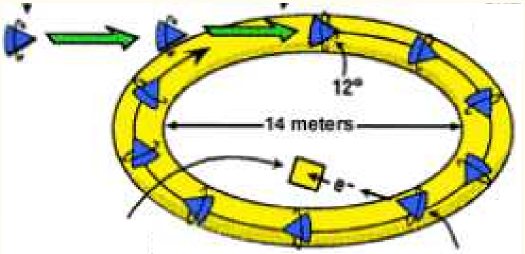
Nature 473, 493 (2011)



The EDM search is a much better probe for New Physics than the MDM, in the case of the electron.

What about the muon dipole moments?

The 3.6 sigma discrepancy persists,



$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 287(80) \times 10^{-11}$$

PRD 86, 095009 (2012)

Suppose again it is due to New Physics. Then the expected EDM is

$$d_\mu \sim \frac{e}{2m_\mu} a_\mu^{\rm NP} \sim 300 \cdot 10^{-11} \frac{e}{200\,{\rm MeV}} \sim 3 \cdot 10^{-22}\, e \cdot {\rm cm}$$

For the muon, the direct bound is much weaker, $|d_\mu| < 1.8 \cdot 10^{-19}\, e \cdot {\rm cm}$

PRD 80, 052008 (2009)

There are ideas/plans to improve the direct bound to 5E-23 ... E-24 (PSI: Kirch et al, FNAL: Roberts et al, J-PARC: Silenko et al).
Very strong motivation!

Muon vs electron: comments

Precision achieved in the studies of magnetic dipole moments

$$\Delta\left(a_e^{\rm SM} - a_e^{\rm exp}\right) \simeq 10^{-12}$$
$$\Delta\left(a_\mu^{\rm SM} - a_\mu^{\rm exp}\right) \simeq 10^{-9}$$

Sensitivity to new physics scales (in general) like the lepton mass squared,

$$a_f^{\rm NP} \sim \frac{m_f^2}{\Lambda^2}$$

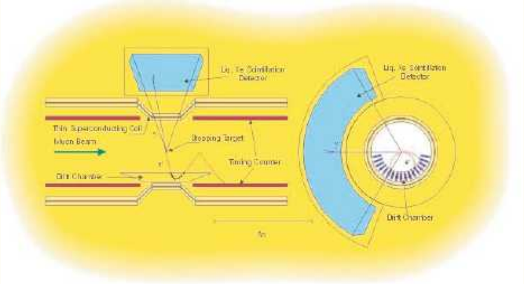
So muon is a more sensitive probe but the electron is becoming relevant,

$$\frac{\Lambda_\mu}{\Lambda_e} \sim \frac{m_\mu}{m_e}\sqrt{\frac{\Delta a_e}{\Delta a_\mu}} \sim 6$$

There are also flavor-off-diagonal dipole moments: muon decay to an electron and photon, $\mu \to e\gamma$

Until recently (MEGA @ Los Alamos): $BR(\mu \to e\gamma) < 10^{-11}$

New bound (MEG @ Paul Scherrer Institute)



$< 5.7 \cdot 10^{-13} \quad (2013)$

This corresponds to the transition dipole moment

$$d_{\mu \to e} \simeq 4 \cdot 10^{-27}\, e \cdot \mathrm{cm}$$

similar to the best electron EDM!

New Physics scales probed by dipole moments

Muon MDM $$d_\mu \sim \frac{e}{2m_\mu} a_\mu^{\rm NP} \sim 3 \cdot 10^{-22}\, e \cdot {\rm cm}$$

Electron EDM $$|d_e| < 10^{-27}\, e \cdot {\rm cm}$$

Muon-electron transition moment $$|d_{\mu\to e}| < 4 \cdot 10^{-27}\, e \cdot {\rm cm}$$

These moments are expected to scale with the New Physics mass like

$$d_f \sim \frac{m_f}{\Lambda^2}$$

The transition moment probes the highest mass scales,

$$\frac{\Lambda_{\mu\to e}}{\Lambda_{\rm eEDM}} \sim \sqrt{\frac{m_\mu}{4m_e}} \simeq 7$$

Bravo MEG!

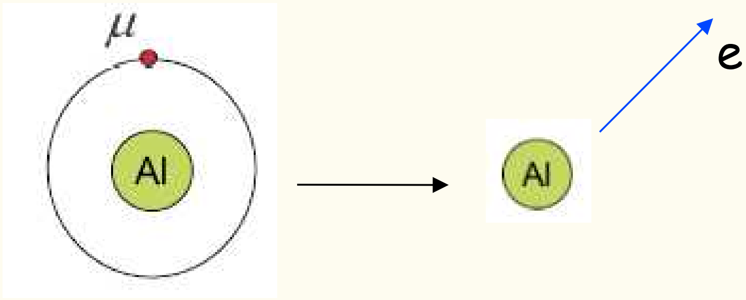
What about non-tensor interactions?

So far, we have only talked about dipole interactions.
There are also vectors and scalars.

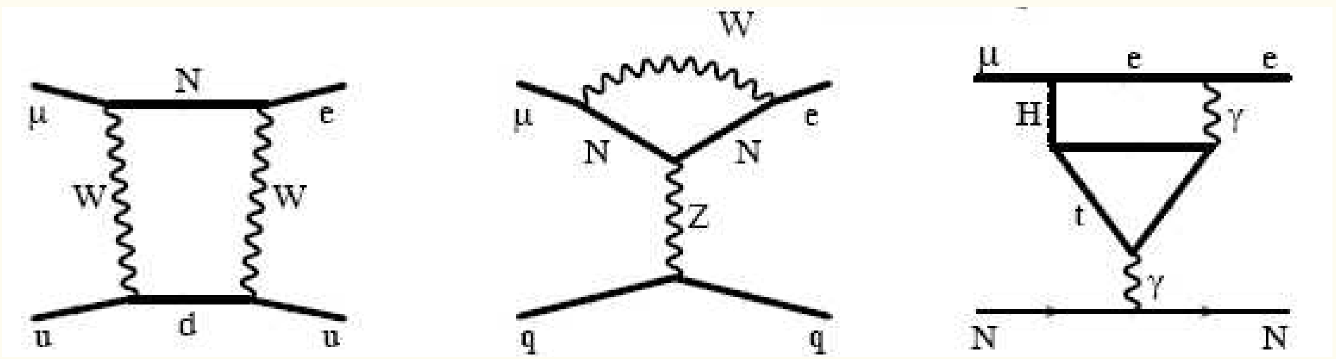
They are not (directly) probed by processes with external photons, by gauge invariance requirements.

New process: muon-electron conversion

(as well as mu --> eee)

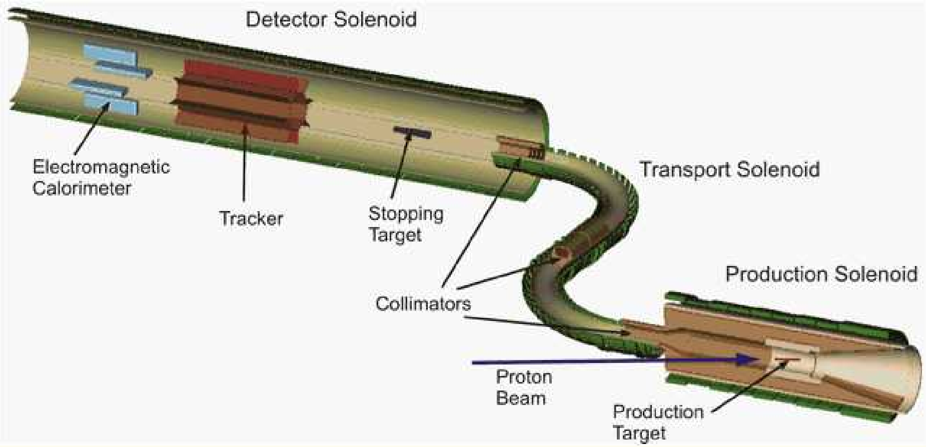


Variety of mechanisms:



Muon-electron conversion

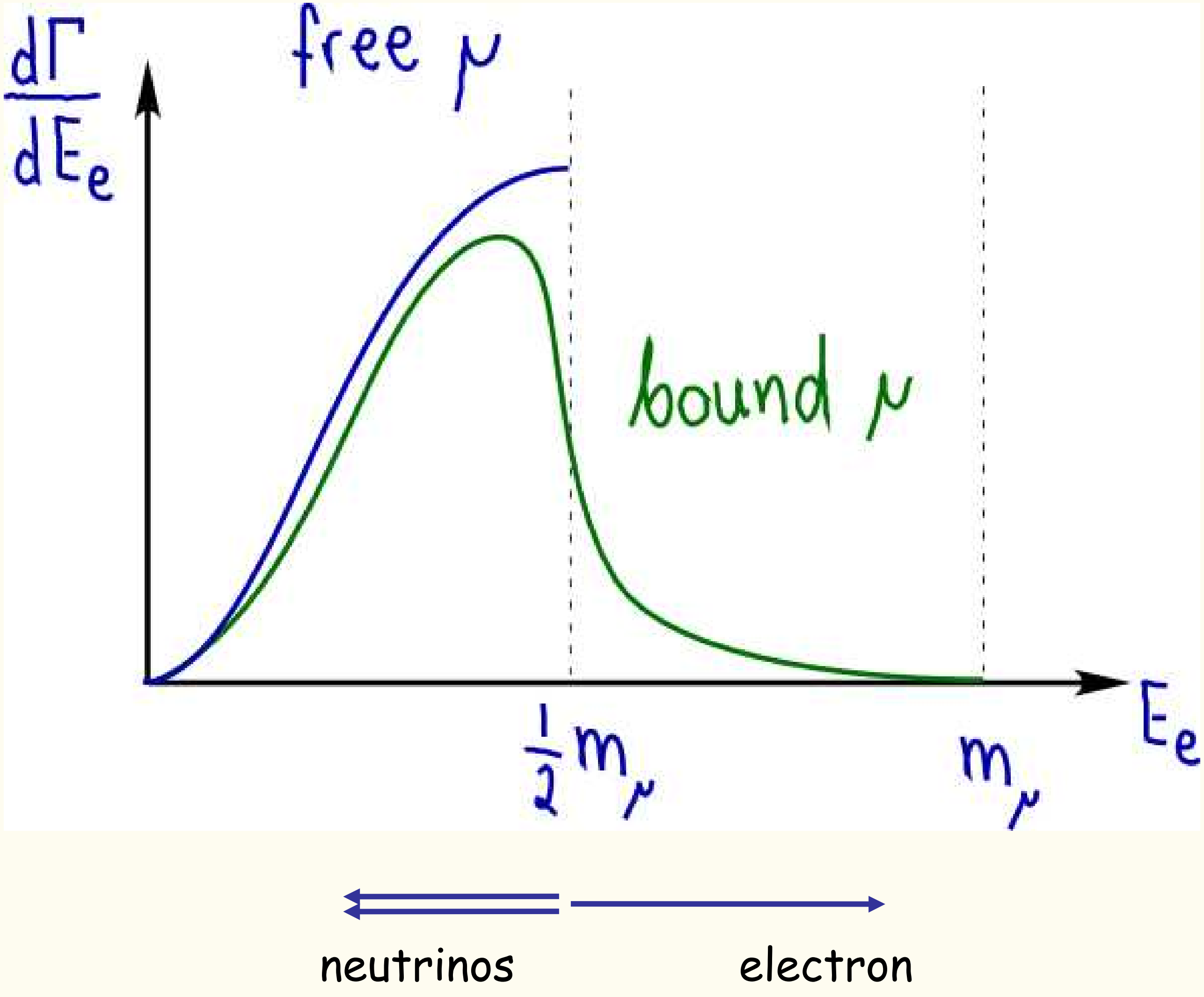
"The best rare process"
No accidental bkgd
(single monochromatic e^-);
10^{-17} sensitivity envisioned



Analogy to fixed-target experiments with a luminosity ~ $10^{50}/\,(\mathrm{cm}^2 \cdot \mathrm{s})$

A year of HL-LHC integrated luminosity collected here every nanosecond!!

Background from the standard muon decay



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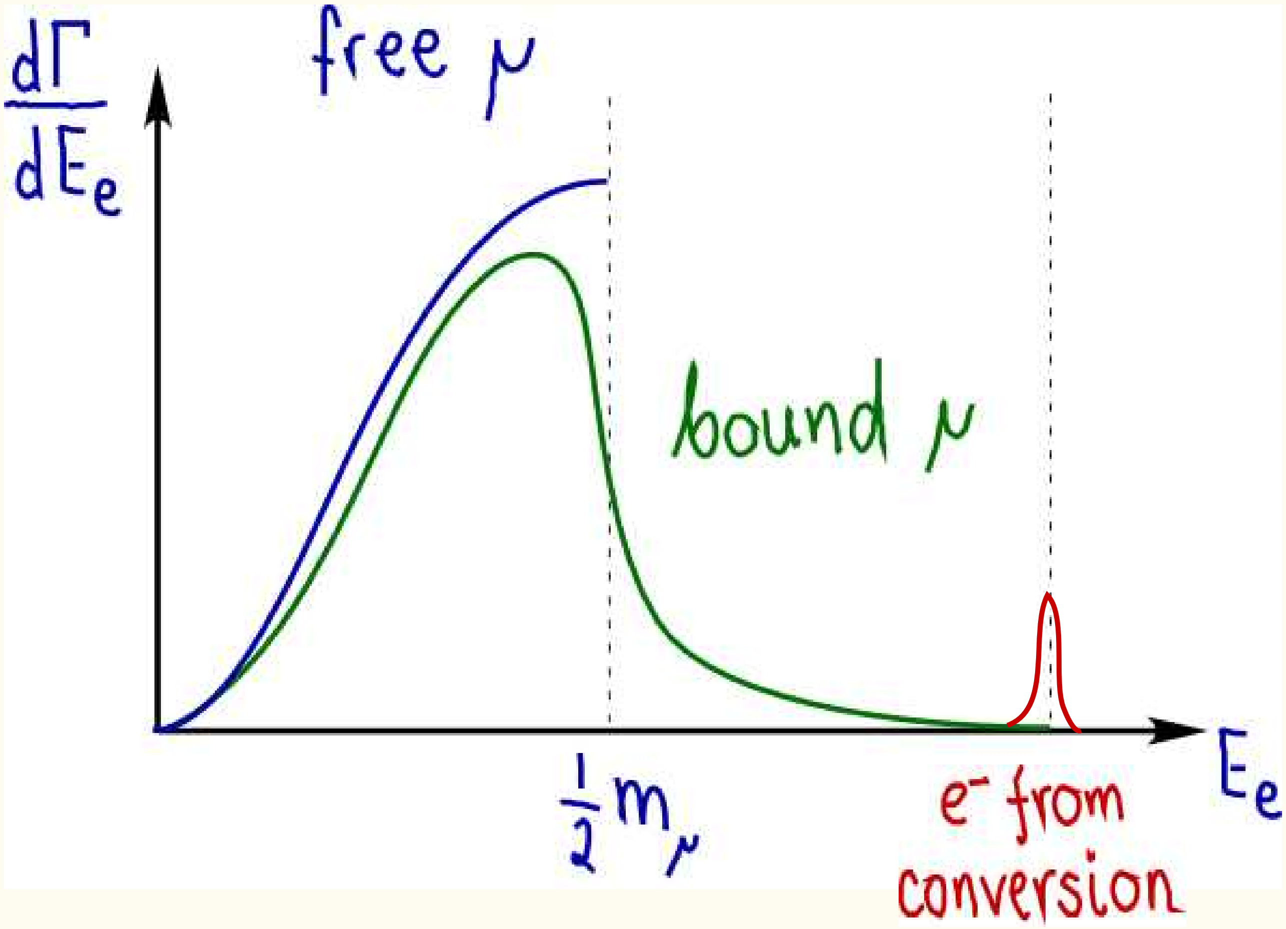
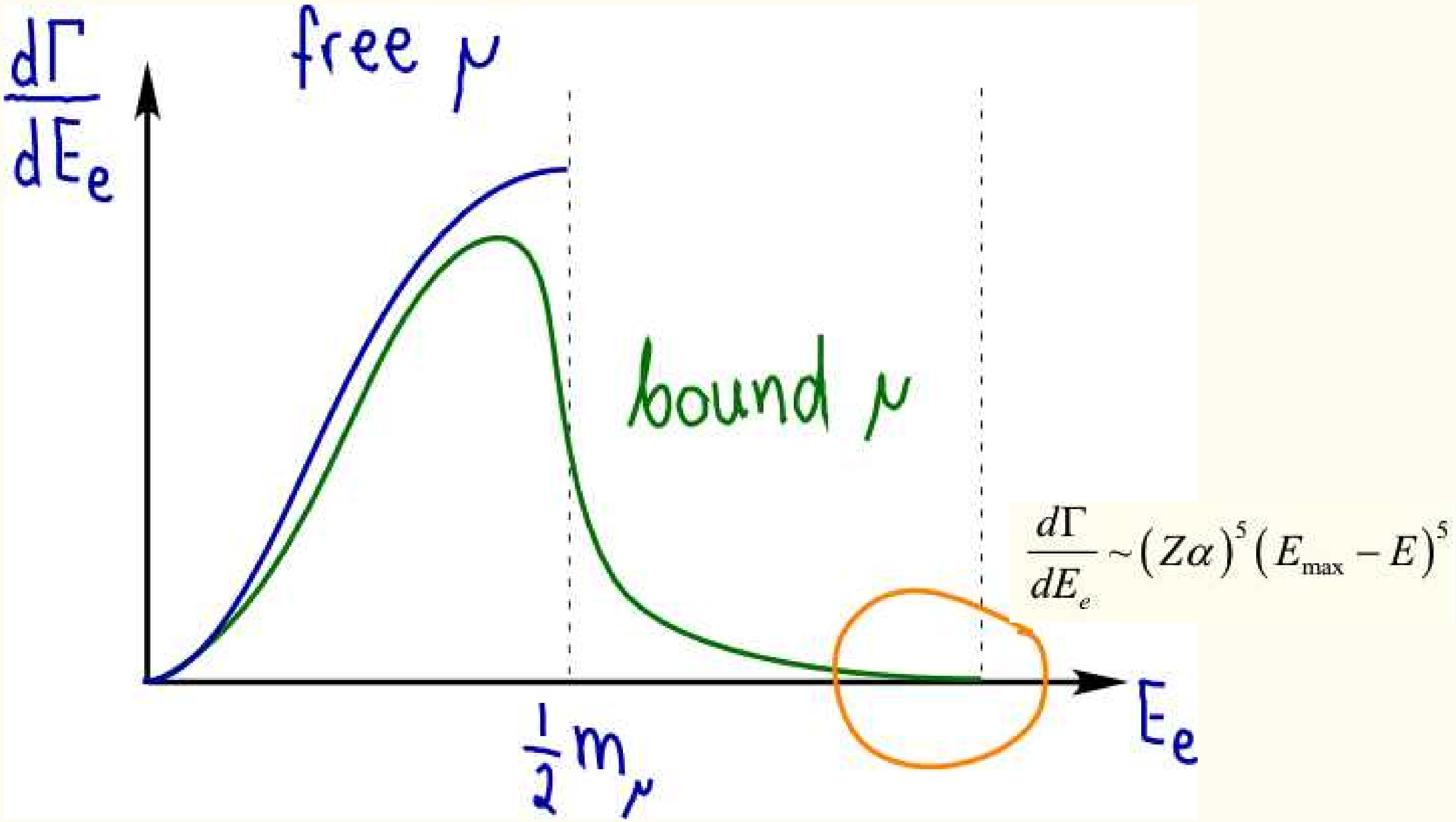


End point spectrum must be well understood



End point spectrum

Previous studies: Shanker & Roy, Hänggi et al., Herzog & Alder

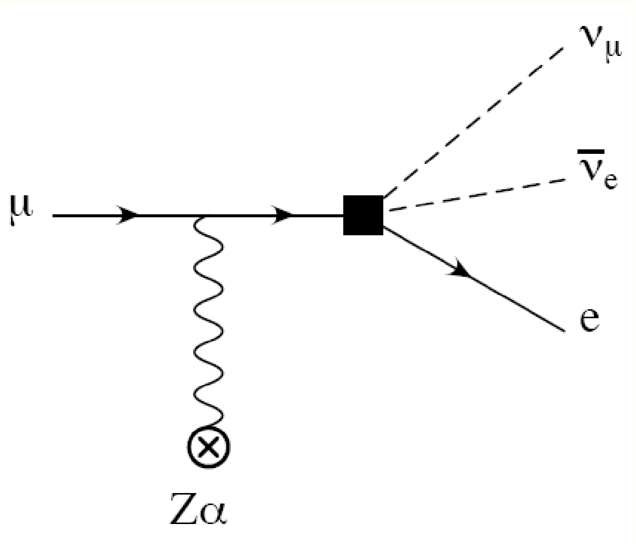
Relativistic muon wave function, nuclear size and recoil, electron final state interactions: all taken into account.

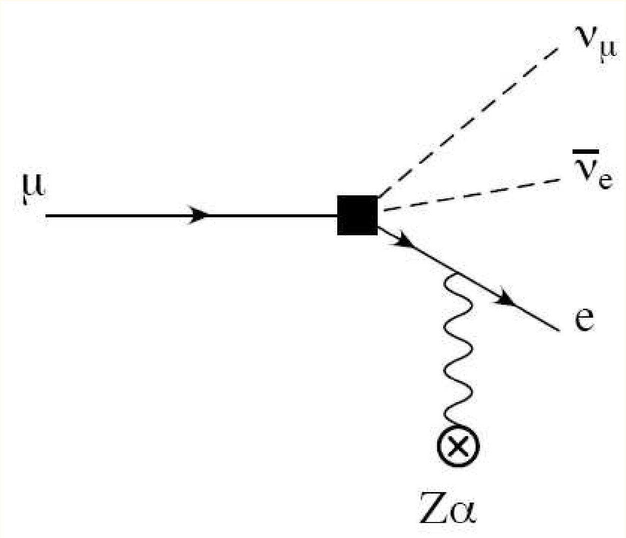
$$N(E_e)\,dE_e \simeq 0.4\cdot 10^{-21}\left(1-\frac{E_e}{E_{max}}\right)^5 dE_e$$

New evaluation: AC, X. Garcia i Tormo, W. J. Marciano PRD84,013006,2011

Planned energy resolution in Mu2e: ~250 keV → 0.22 background events.

How can the electron get muon's whole energy?





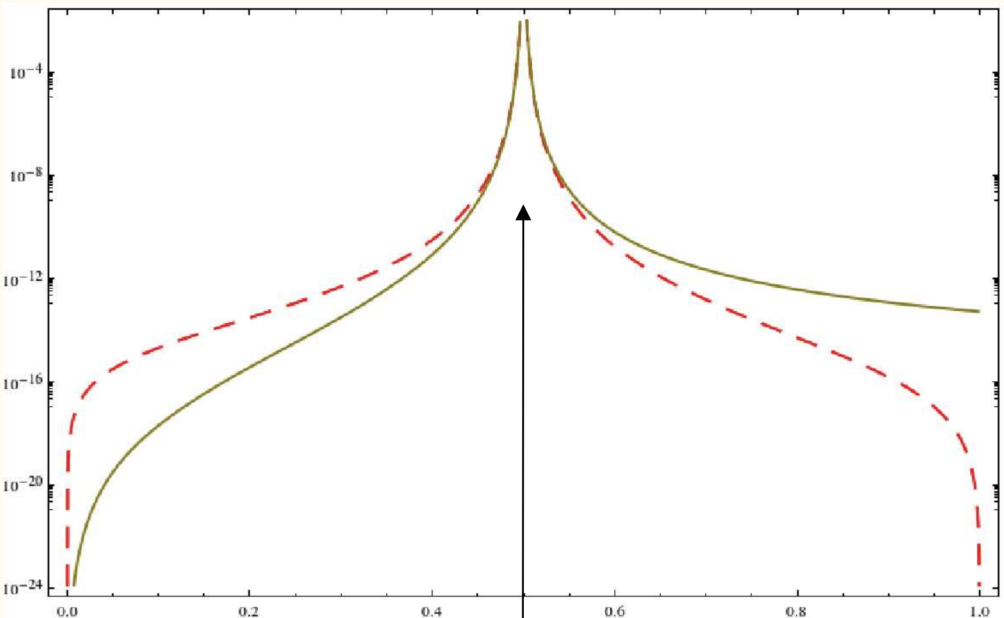
Neutrinos get no energy;
The nucleus balances electron's momentum, takes no energy.
Near the end point:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_e} \sim |\psi(0)|^2 (Z\alpha)^2 \frac{\mathrm{d}^3\nu_e}{\nu_e}\frac{\mathrm{d}^3\nu_\mu}{\nu_\mu}\delta\left(E_{\max} - E_e - \nu_e - \nu_\mu\right) \mathrm{Tr} \ldots \not{\nu}_e \ldots \not{\nu}_\mu$$

$$\sim (Z\alpha)^5 \left(E_{\max} - E_e\right)^5$$

Results: electron spectrum in $\mu \rightarrow e + J$

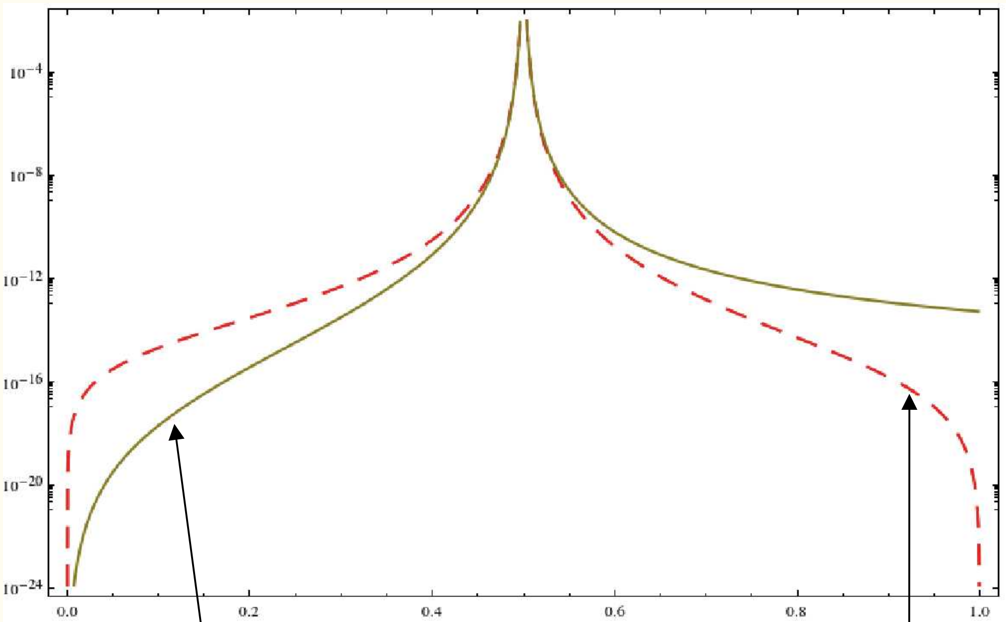
Robert Szafron

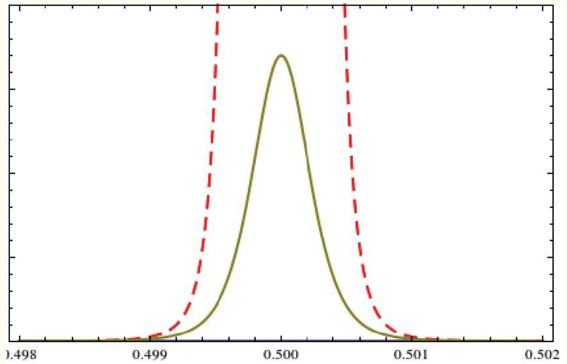


without binding effects,
the electron spectrum is
monochromatic,
concentrated here
at half muon mass

Results: electron spectrum in μ→ e+J

Robert Szafron

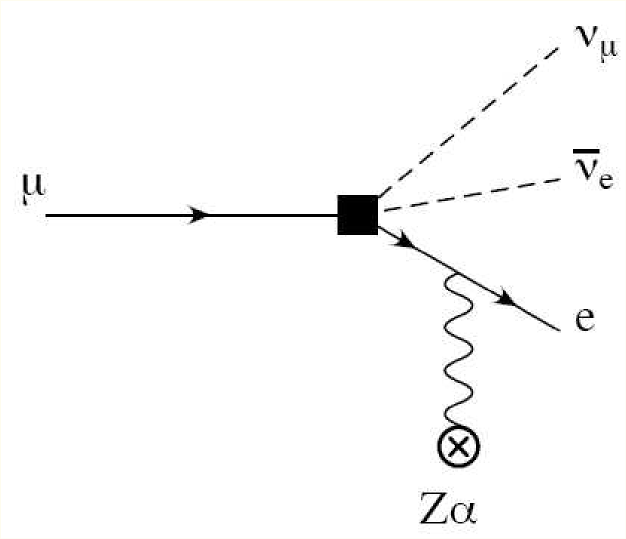




smearing due to
muon's motion.
Dominates in the center.

expansion
in Z*alpha
Correct far
from the center

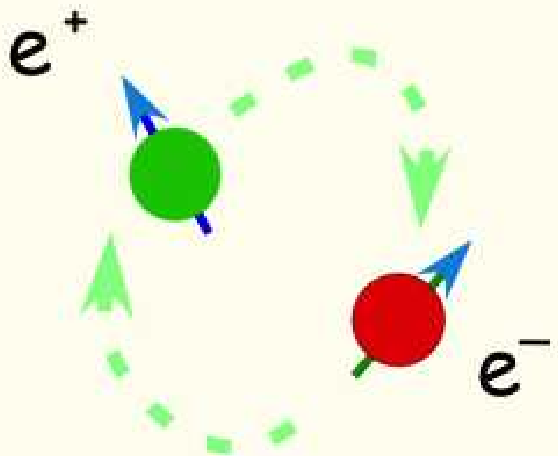
Next step: radiative corrections to the electron spectrum

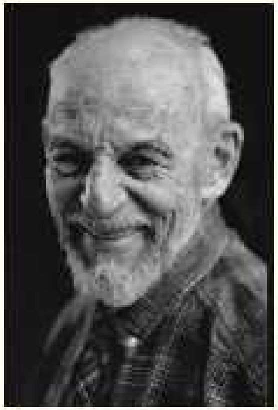


Competing effects:
- vacuum polarization in the hard photon; and
- self-energy and real radiation

Ultimate goal: smooth matching of all energy regions, from the bound electron at low energy to the end-point.

Positronium





Martin Deutsch
1917 - 2002

Very similar to hydrogen, except
- no hadronic nucleus
- annihilation
- reduced mass reduced $m_e \to \frac{m_e}{2}$

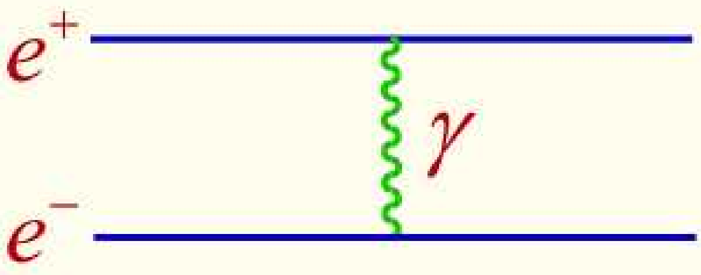
Two spin states:
singlet (para-Ps; short-lived, 0.1 ns)
triplet (ortho-Ps; long-lived, ~150 ns)

All properties can be described by QED, using one parameter: $\alpha = \frac{1}{137.036}$

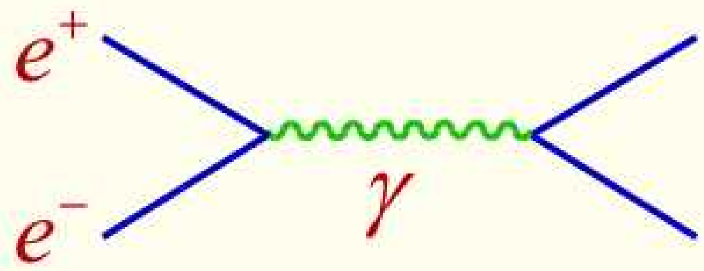
Positronium spectrum: discrepancy with QED

Tree-level QED prediction for the hyperfine splitting (HFS)

Recoil effect



Annihilation effect

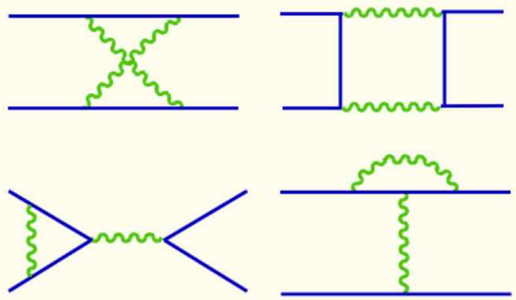


$$\gamma_\mu \otimes \gamma^\mu \to 1 \otimes 1 + \sigma \otimes \sigma$$

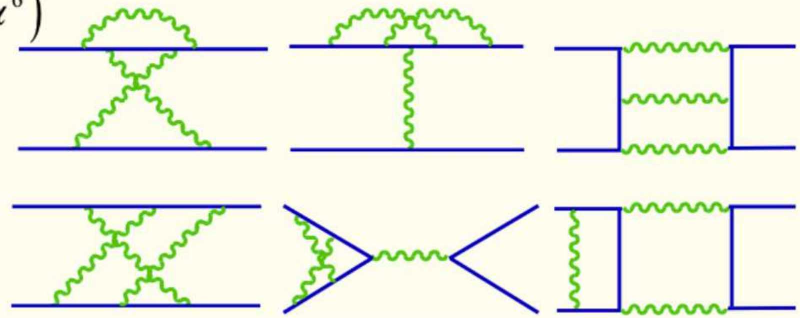
$$\Delta \nu_{\rm HFS} = \frac{7}{12} m_e \alpha^4 \simeq 204 \text{ GHz}$$

Loop corrections to the HFS

$O(\alpha^5)$



$O(\alpha^6)$



$$-\frac{m_e\alpha^5}{\pi}\left(\frac{8}{9}+\frac{\ln 2}{2}\right)$$

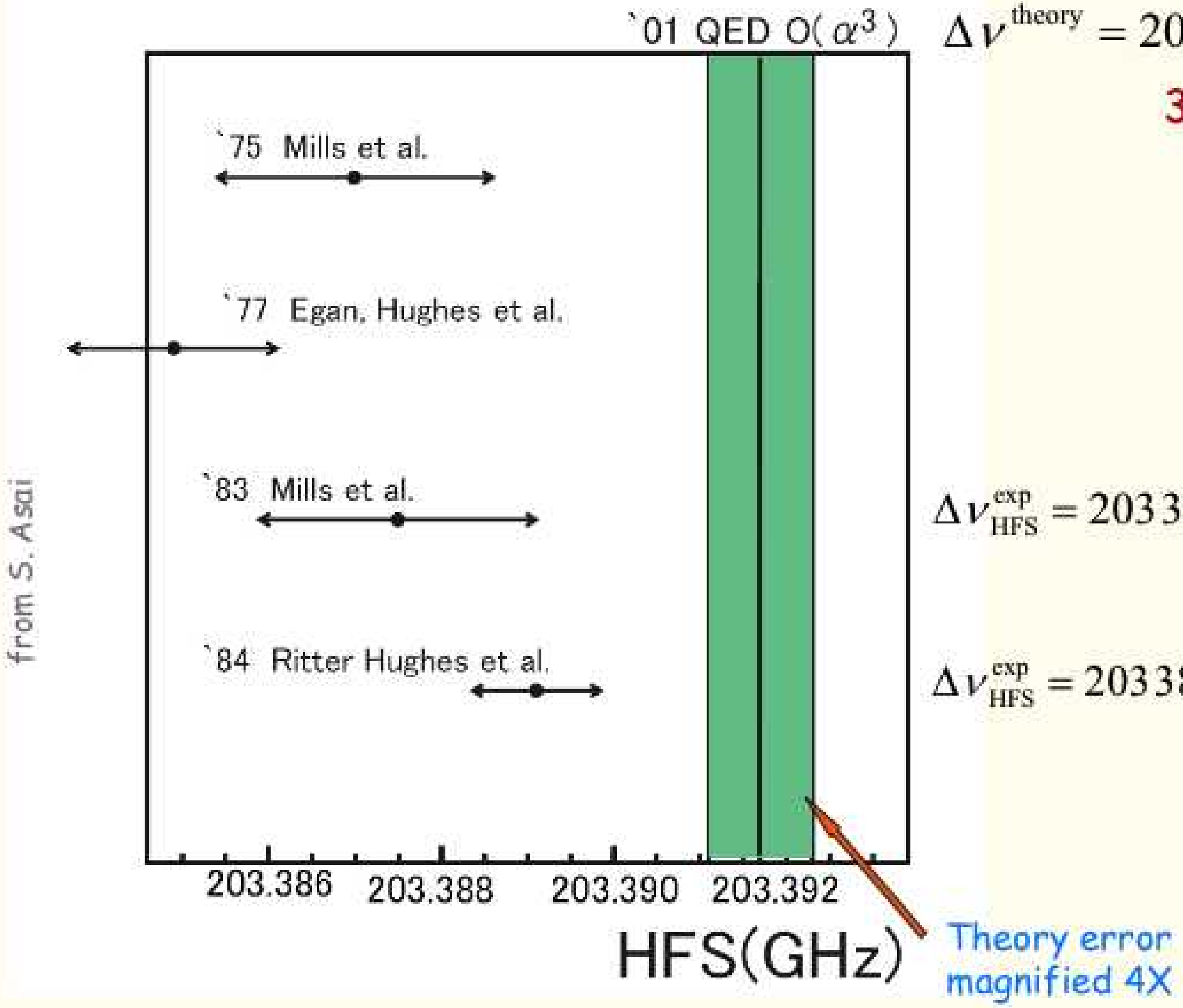
$$\simeq -1005.5\ \text{MHz} \rightarrow -0.5\%$$

$$\frac{m_e\alpha^6}{\pi^2}\left[\frac{1367}{648}-\frac{5197}{3456}\pi^2+\left(\frac{1}{2}+\frac{221}{144}\pi^2\right)\ln 2-\frac{53}{32}\zeta(3)+\frac{5}{24}\pi^2\ln\frac{1}{\alpha}\right]$$

$$\simeq 11.8\ \text{MHz} \rightarrow 0.006\%$$

(Experimental error $\simeq 0.7$ MHz)

HFS theory vs. measurements



$$\Delta\nu^{theory} = 203391.69(16)\ \text{MHz}$$

3.4σ off!

Kniehl & Penin

$$\Delta\nu^{exp}_{HFS} = 203387.5(1.6)\ \text{MHz}$$

$$\Delta\nu^{exp}_{HFS} = 203389.10(74)\ \text{MHz}$$

Previous experiments: used para-ortho mixing

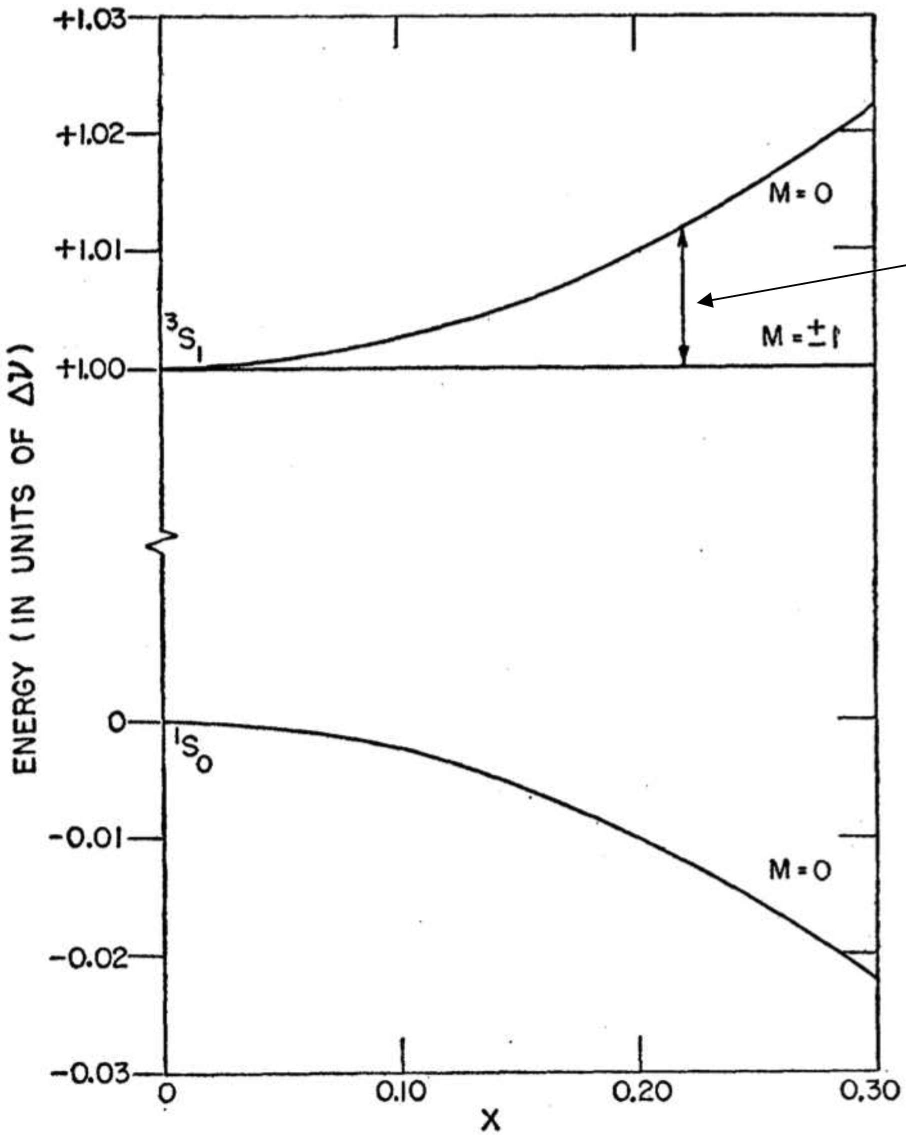
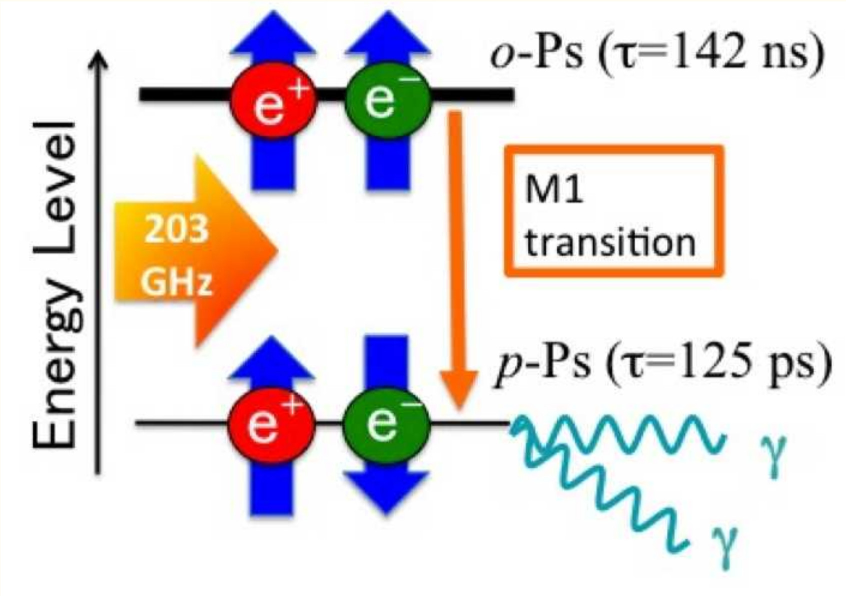


FIG. 1. Zeeman energy levels of positronium in its ground $n=1$

This splitting proportional to electron's g-factor, in the bound state

New experiment aims at the direct transition

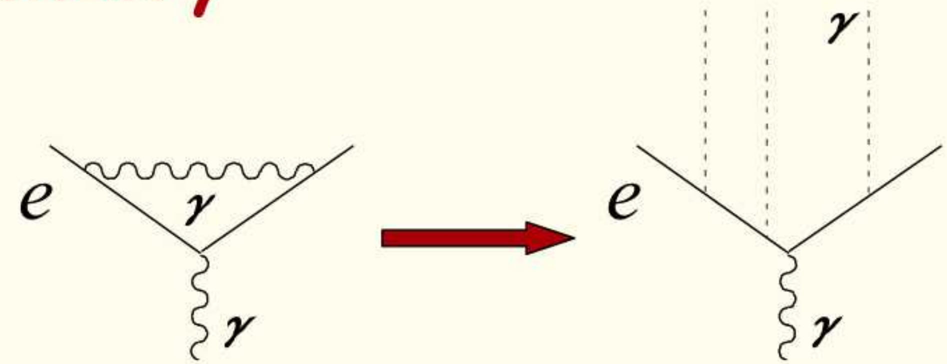
University of Tokyo



A direct transition between ortho- and para-positronium
has very recently been observed for the first time: PRL 108, 253401 (2012).

Goal: to reach a ppm precision ~ 0.2 MHz

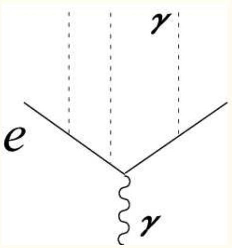
Bound-electron g-2: theory

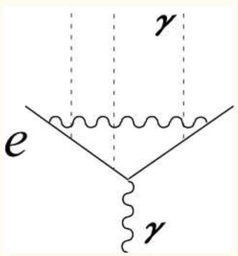


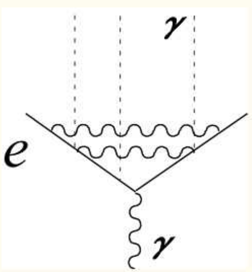
$$g = 2 - \frac{2\,(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \ldots = \frac{2}{3}\left[1 + 2\sqrt{1-(Z\alpha)^2}\right]$$

Breit 1928 – Dirac theory

Bound-electron g-2: theory







$$g = 2 - \frac{2\,(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \ldots$$

$$+\frac{\alpha}{\pi}\left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\,(a_{41}\ln Z\alpha + a_{40}) + \ldots\right]$$

$$+\left(\frac{\alpha}{\pi}\right)^2\left[-0.65..\left(1 + \frac{(Z\alpha)^2}{6}\right) + (Z\alpha)^4\,(b_{41}\ln Z\alpha + b_{40}) + \ldots\right]$$

two-loop corrections

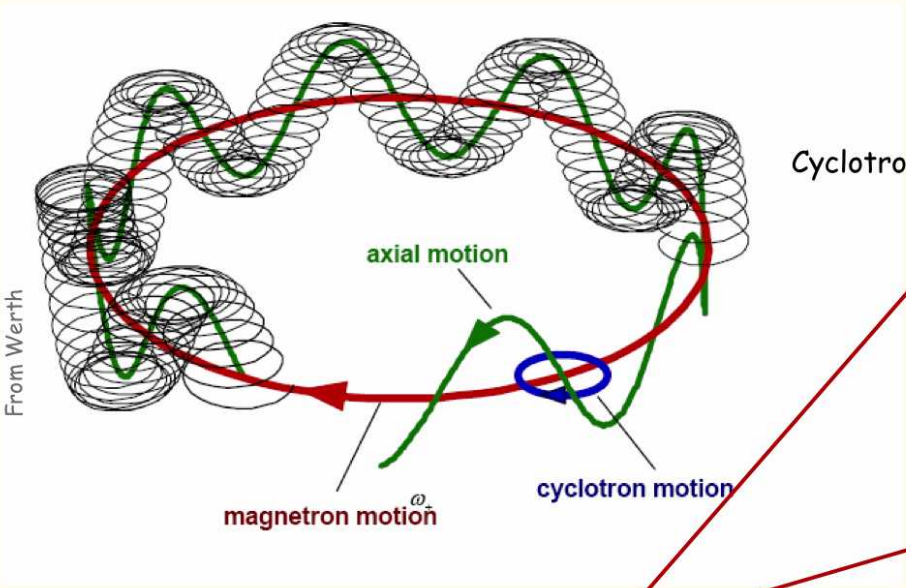
$$b_{41} = \frac{28}{9}$$

$$b_{40} = -16.4$$

Pachucki, Jentschura, Yerokhin, AC

Bound g factor and the electron mass determination

Motion in a Penning trap



Spin precession (Larmor) frequency

$$\hbar\omega_L = g\frac{e}{2m}\vec{s}\cdot\vec{B}$$

Cyclotron frequency

$$\hbar\omega_c = \frac{q}{2M}\vec{L}\cdot\vec{B}$$

$$\frac{m}{M} = \frac{g\omega_c e}{2\omega_L q}$$

This g factor is modified
by the electron binding to the nucleus

$$m_e\left({}^{12}C^{5+}\right) = 0.000\,548\,579\,909\,31(29)_{\rm exp}(1)_{\rm th}\;u$$

Theoretical error: negligible

Summary

Low energy experiments are excellent probes for New Physics.

Pushing the limits of experimental and theoretical techniques.

Exciting future prospects at PSI, Fermilab, J-PARC, among others.

Goal: combine low-energy probes with the LHC;
leave New Physics no space to escape!

Summary

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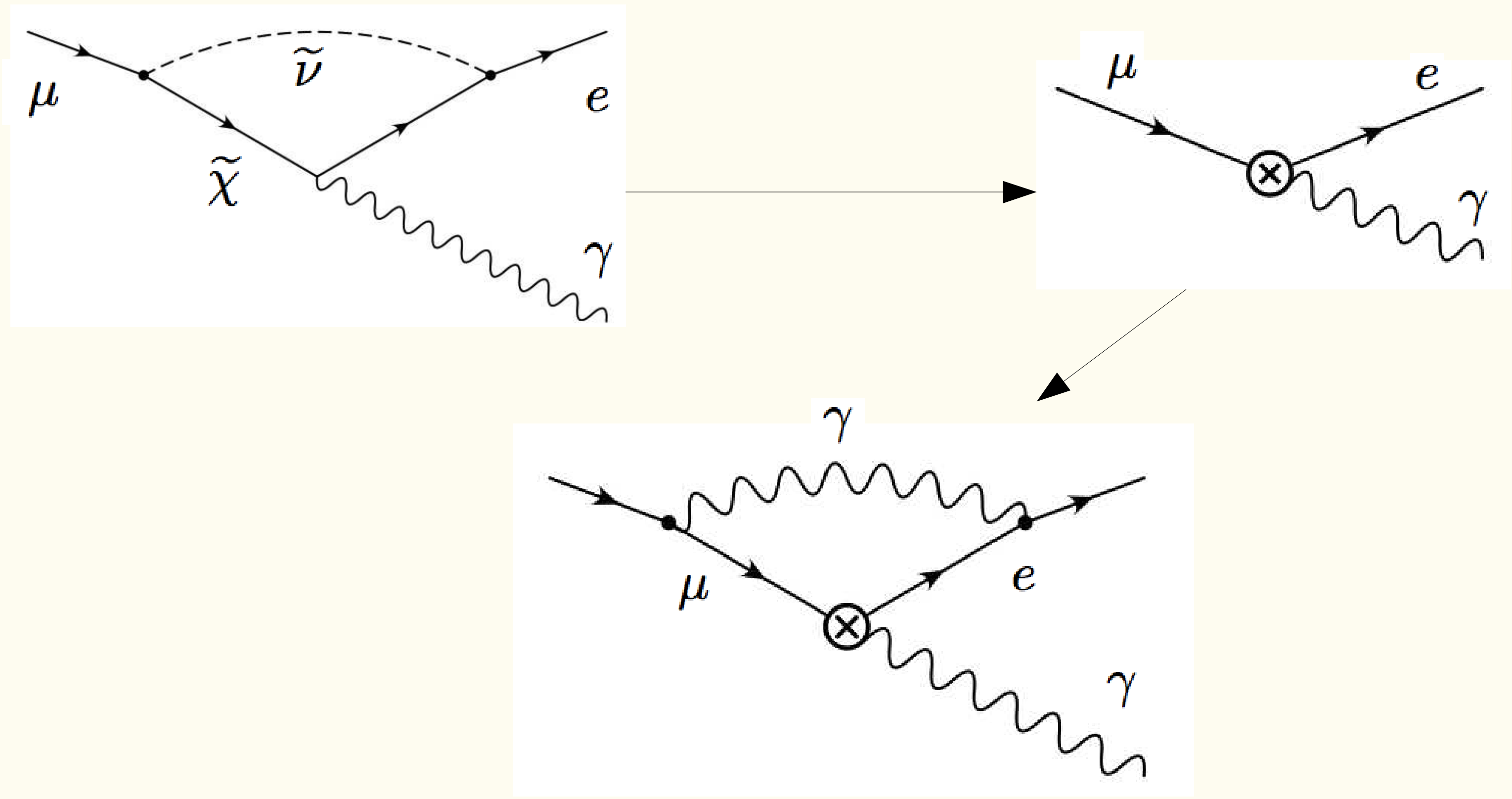
Extra slides

An exceptional radiative correction



Unusual QED suppression ~15% (large log of the new physics scale Λ)

$$\Gamma(\mu \to e\gamma) \simeq \left(1 - \frac{8\alpha}{\pi} \ln \frac{\Lambda}{m_\mu}\right) \Gamma^{(0)}(\mu \to e\gamma)$$

Phys. Rev. D 65, 113004