The size of the proton
from
muonic hydrogen

Aldo Antognini, ETH

for the

CREMA collaboration

The size of the proton
from
muonic hydrogen

Measure $\Delta E(2S-2P)$
$\rightarrow\ r_\mathrm{p}$ with $\delta r_\mathrm{p} = 4 \times 10^{-19}$ m

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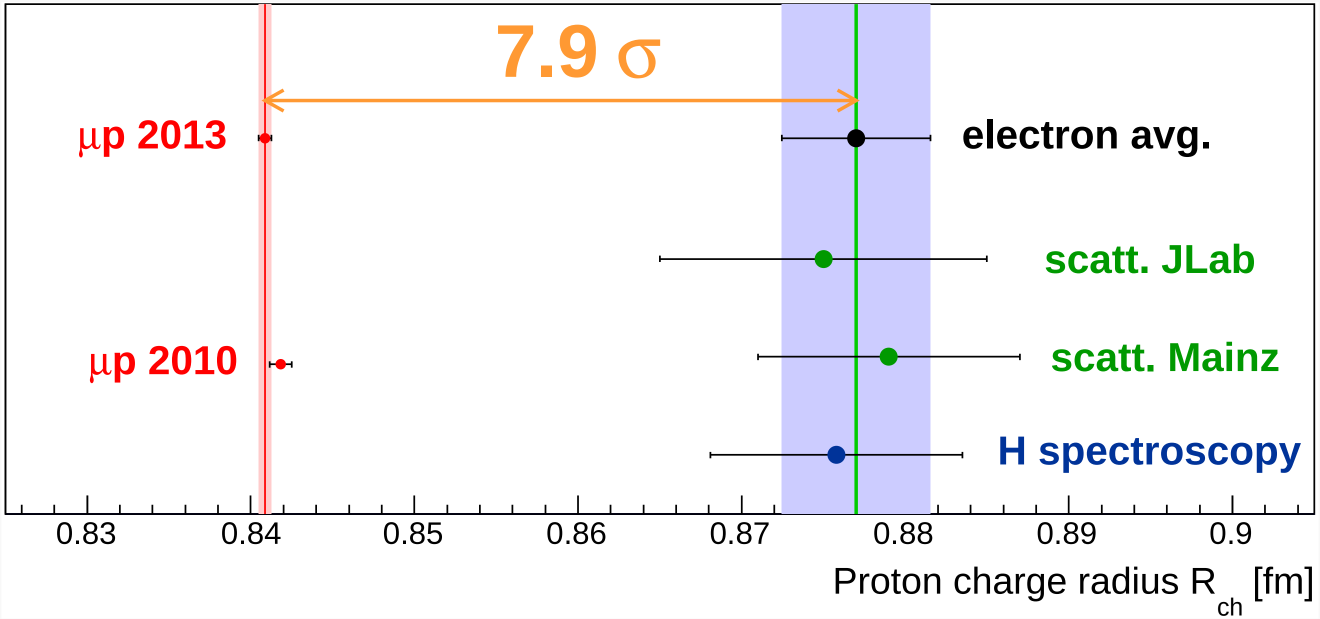
The proton radii puzzle

3 ways to the proton radius

e-p scattering
H precision laser spectroscopy
μp laser spectroscopy

Our value is 20 times more precise,
but at large discrepancy



Proton radius from muonic hydrogen

- Measure $\Delta E^{\mathrm{exp}}(2P-2S)$ in $\mu\mathrm{p}$
  using laser spectroscopy with $u_r = 10^{-5} \leftrightarrow 0.5\ \mathrm{GHz} = \Gamma/20$

- Compute theoertical prediction
  using bound-state QED

$$\Delta E^{\mathrm{th}}(2P-2S) = 206.0668(25) - 5.2275(10)\, r_{\mathrm{p}}^2 \quad [\mathrm{meV}]$$

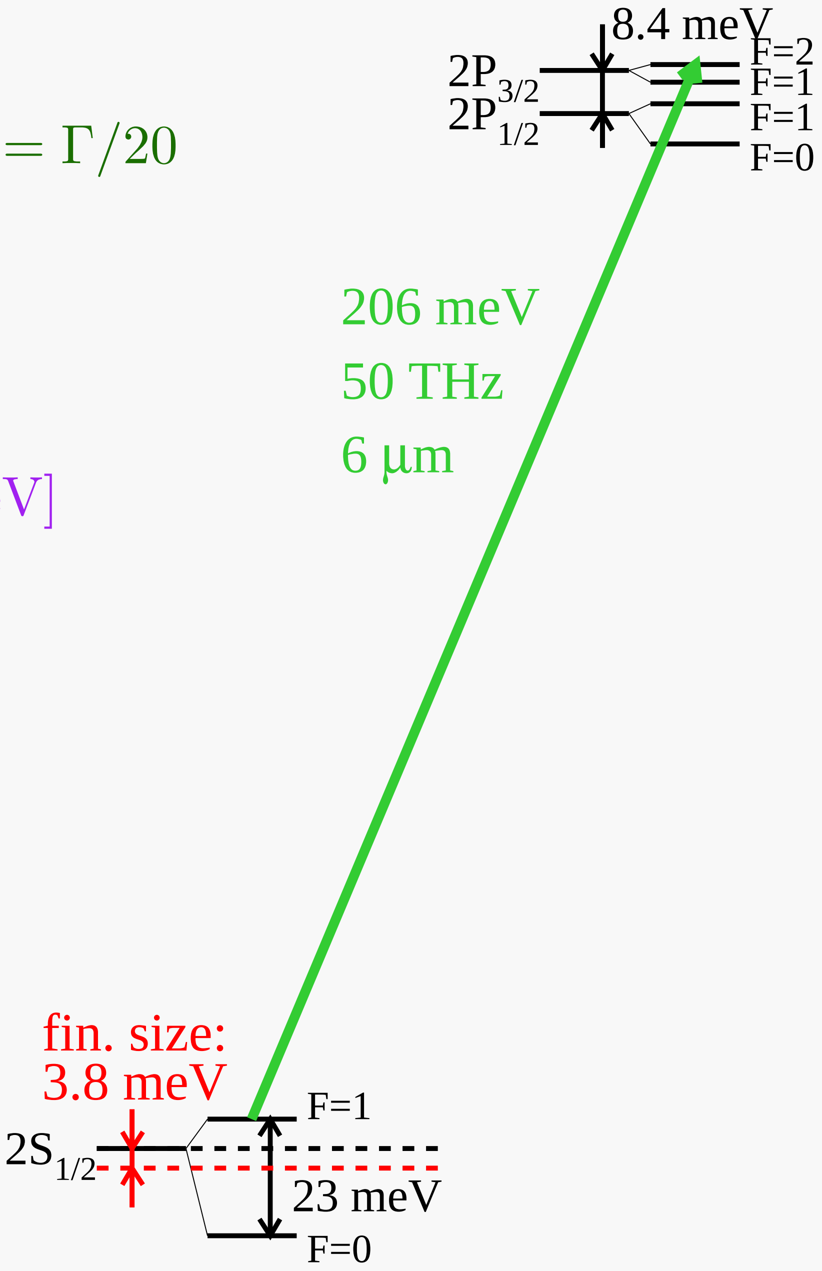
Comparing theory with experiment

$$\Longrightarrow r_{\mathrm{p}} \text{ with } u_r \approx 5 \times 10^{-4}$$

$$\Delta E = \Delta E_{\mathrm{QED}} + \Delta E_{\mathrm{fs}}$$

$$\begin{aligned} \Delta E_{\mathrm{fs}}^{(0)} &= \frac{2\pi(Z\alpha)}{3} \langle r_{\mathrm{p}}^2 \rangle\, |\Psi_n(0)|^2 \\ &= \frac{2(Z\alpha)^4}{3n^3} m_r^3 \langle r_{\mathrm{p}}^2 \rangle\, \delta_{l0} \end{aligned}$$

$$m_\mu \approx 200 m_e$$



Principle of the μp Lamb shift experiment

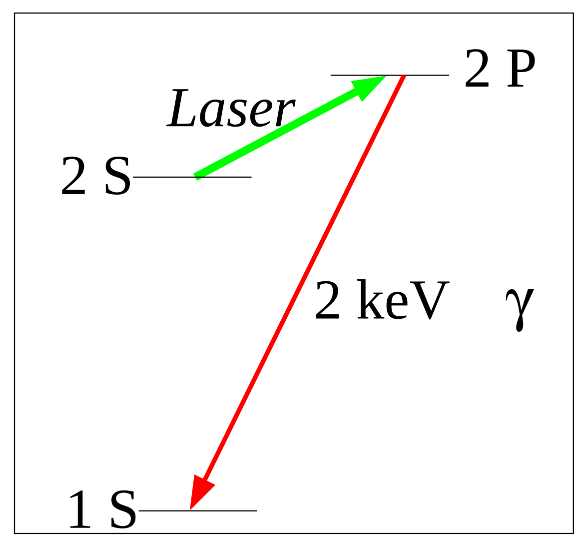
- Produce many μ^-

PSI accelerator

- Stop μ^- in 1 mbar H_2 gas
  → μp formation
  (1% in the 2S-state with 1μs lifetime)

Dedicated low-energy μ^- beam line

- Fire laser at $\lambda = 6\mu$m
  → to induce $\mu\mathrm{p}(2S) \rightarrow \mu\mathrm{p}(2P)$ transition



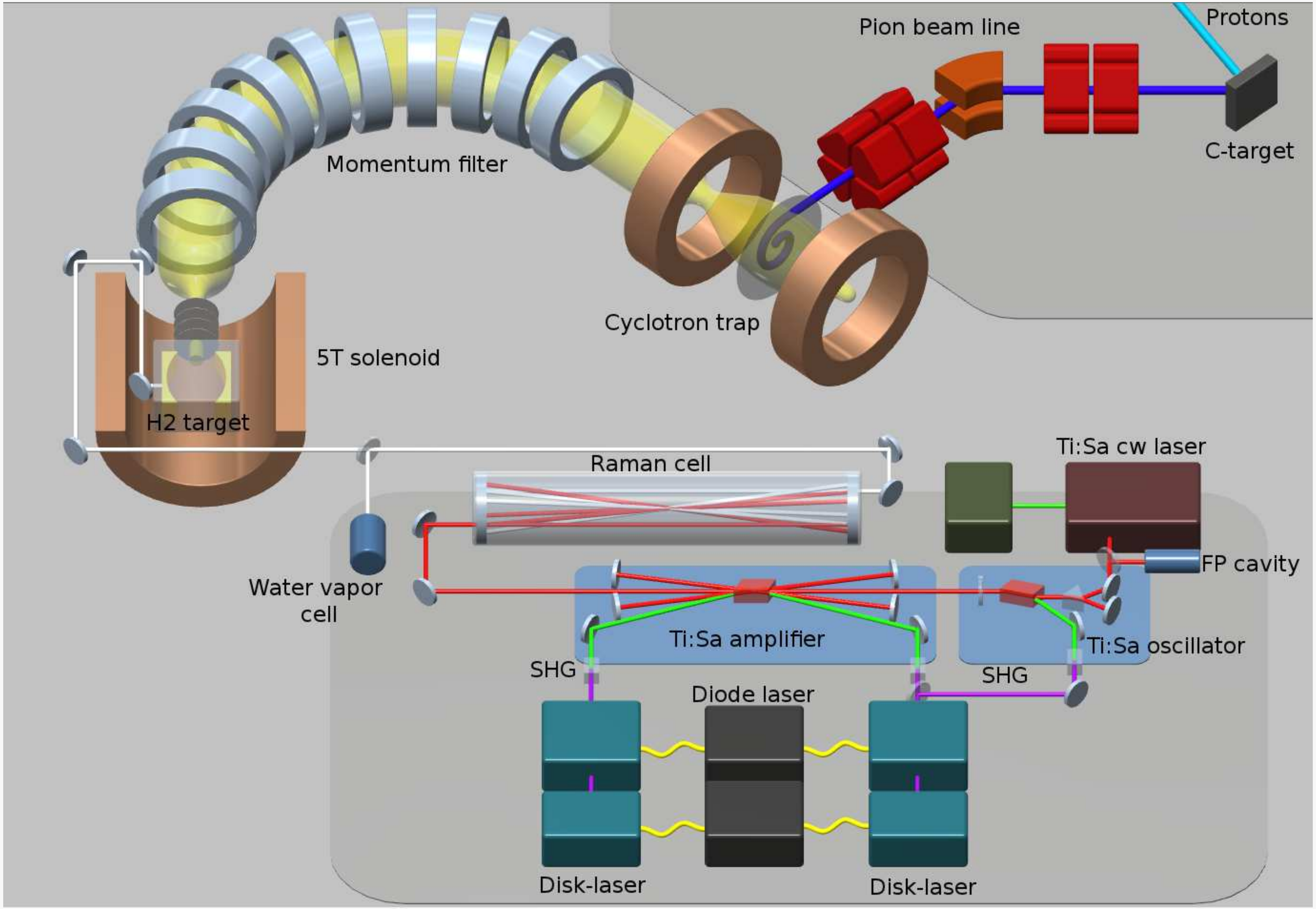
Dedicated laser system
with “strange” requirements

- If laser resonant
  → observe 2 keV x-rays

2 keV x-ray detectors

The μp Lamb shift setup

p-beam
(1 MW)
↓
π (MeV)
↓
μ (MeV)
↓
μ (keV)
↓
μp(2S)
↓
Laser
↓
X-ray



940 nm
(2 kW)
↓
1030 nm
↓
515 nm
↓
708 nm
↓
1.0 μm
↓
1.6 μm
↓
6.0 μm

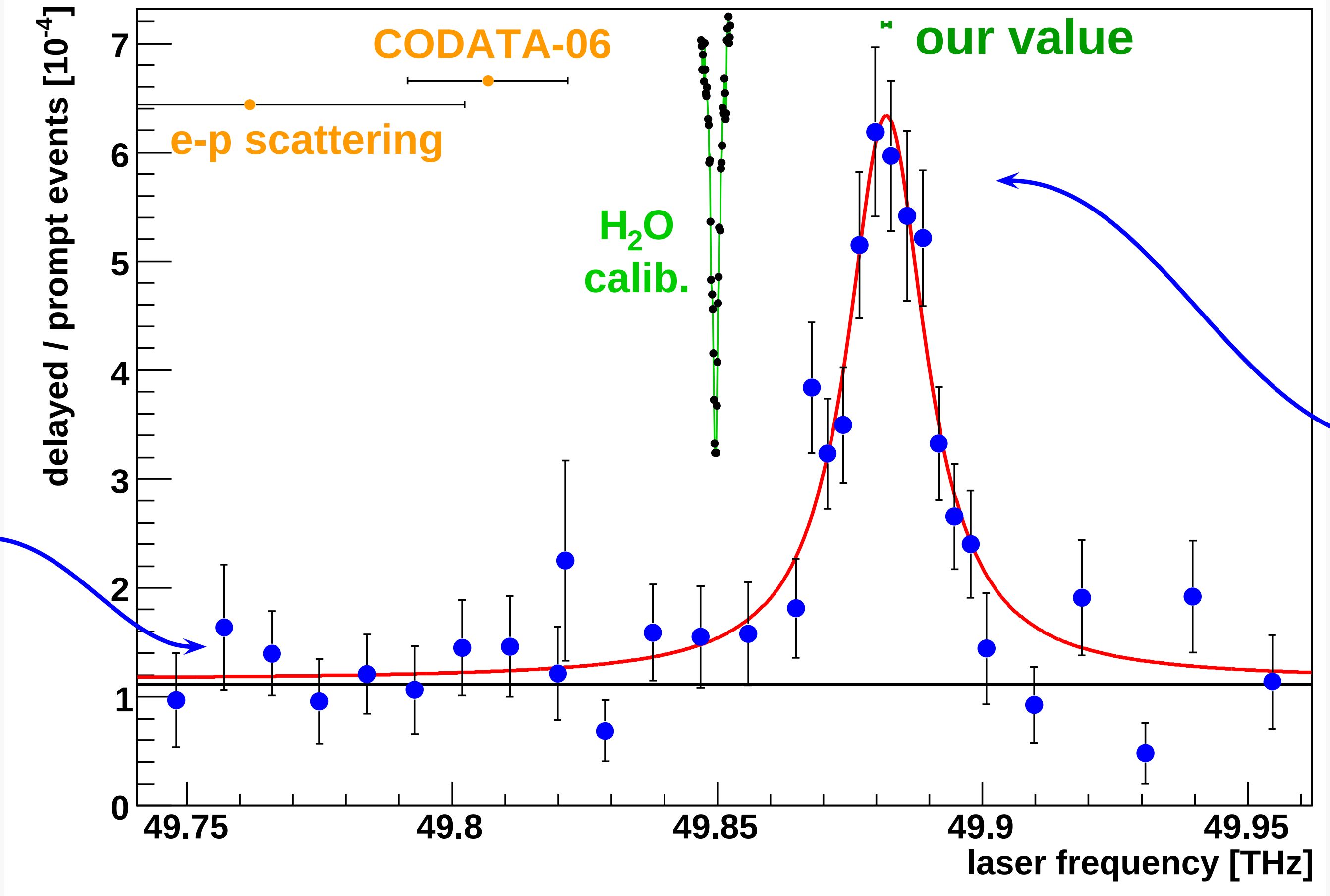
The first muonic hydrogen resonance (2010)

Predictions were off by (discrepancy):

$5.0\,\sigma \;\leftrightarrow\; \sim 75\ \mathrm{GHz} \;\leftrightarrow\; \delta\nu/\nu = 1.5 \times 10^{-3}$

Rate:

1 event/hour



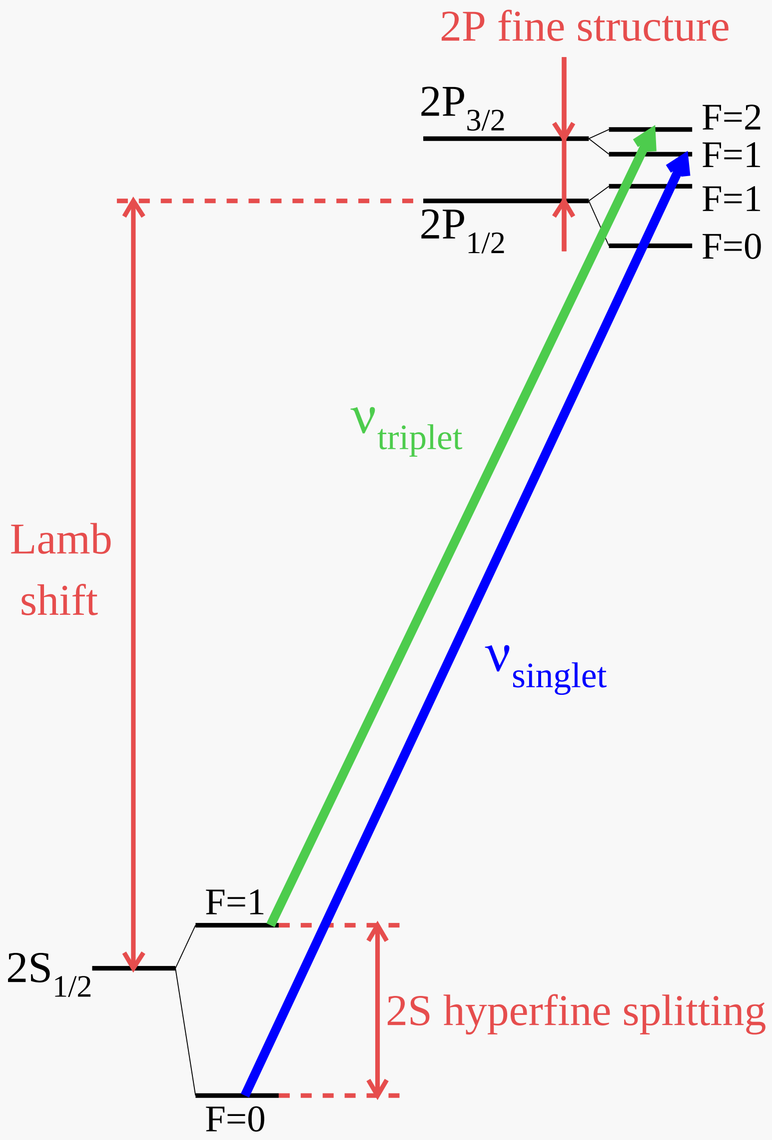
Rate:

6 events/hour

We measured 2 transitions in μp

and 3 in μd (to be published)

We have measured two transitions in μp



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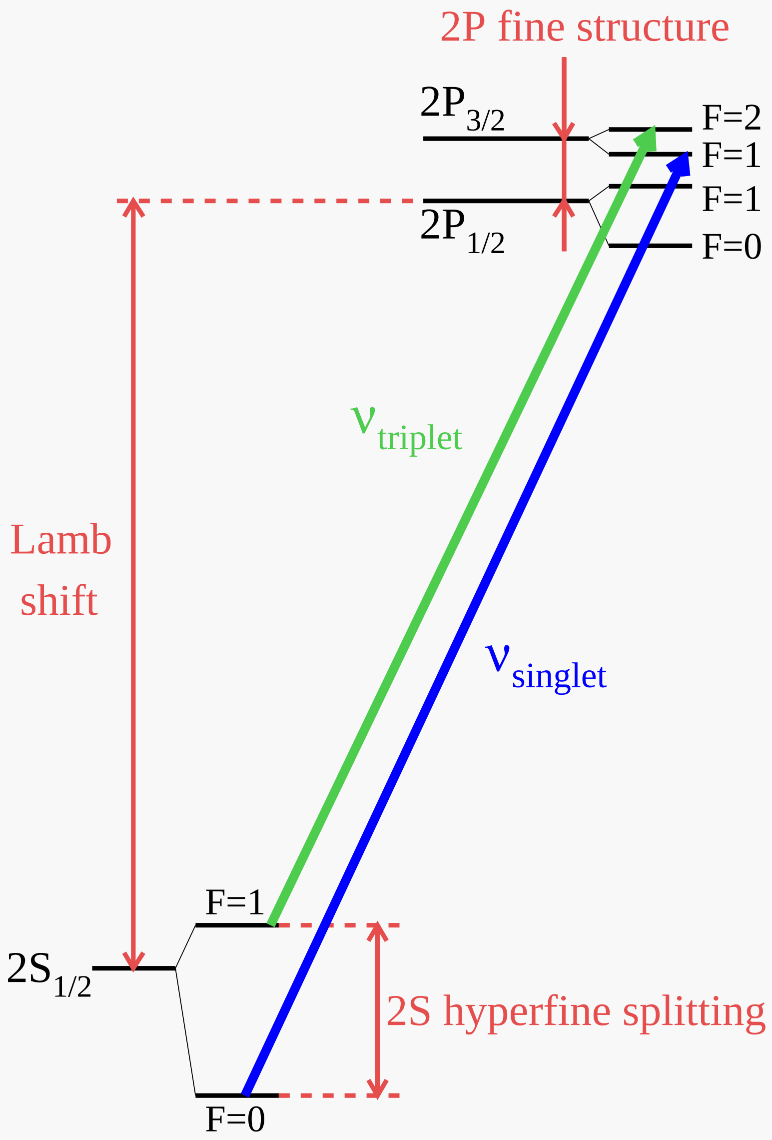

- Considering the two measurements separately

Two independent determinations of r_p

$(\nu_t \rightarrow r_\mathrm{p}\ ,\ \nu_s \rightarrow r_\mathrm{p})$ Consistent results !!!

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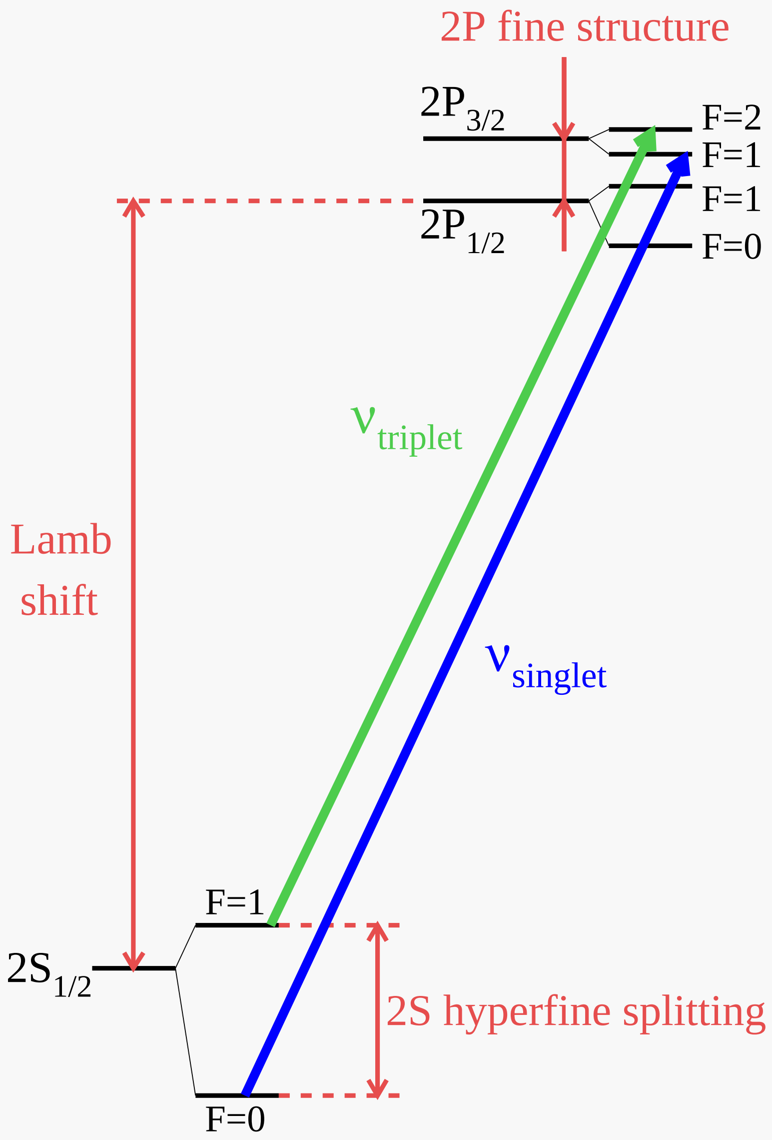

- Considering the two measurements separately

Two independent determinations of r_p

$(\nu_t \rightarrow r_p \ , \ \nu_s \rightarrow r_p)$ **Consistent results !!!**

- Combining the two measurements

Two measurements → determine two parameters

$\nu_t, \nu_s \rightarrow \Delta E_L, \Delta E_{HFS} \rightarrow \boxed{r_p, r_Z}$

$$r_Z = \int d^3r_1 \, d^3r_2 \, \rho_E(r_1)\rho_M(r_2)|r_1 - r_2|$$

$$\begin{array}{rcll} \frac{3}{4}\nu_t + \frac{1}{4}\nu_s & = & \Delta E_L(r_p) & + 8.8123 \ \text{meV} \\ \nu_s - \nu_t & = & \Delta E_{HFS}(r_Z) & - 3.2480 \ \text{meV} \end{array}$$

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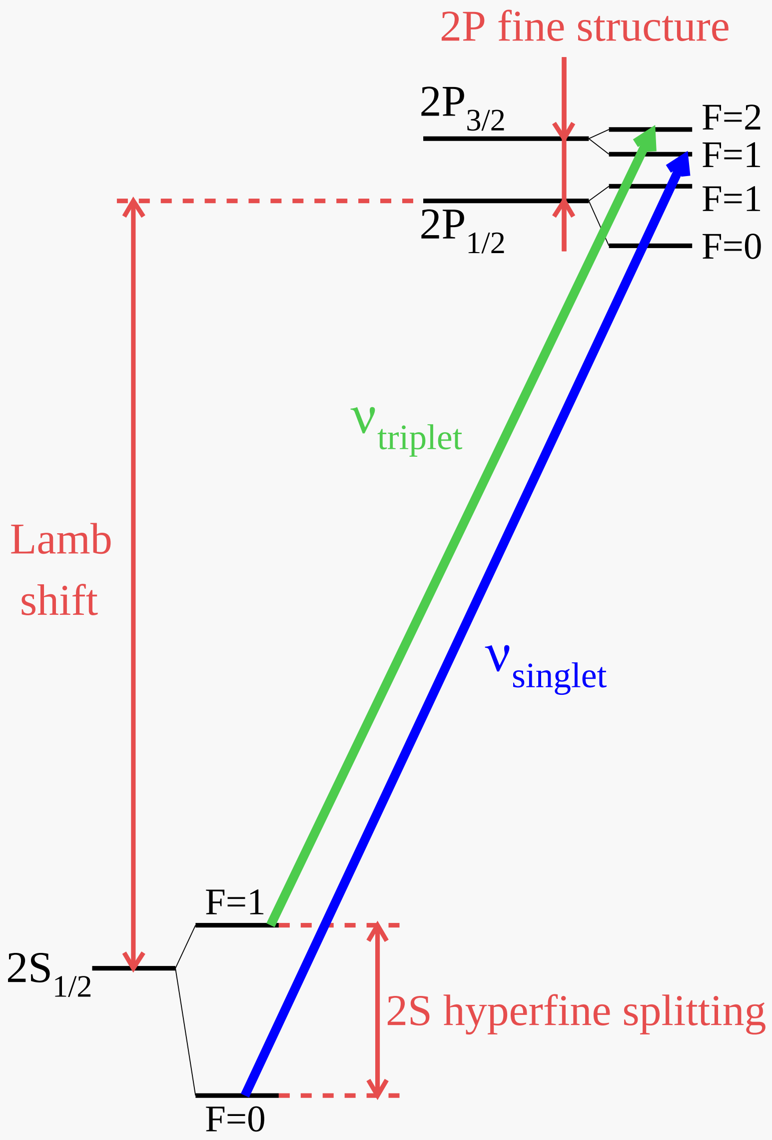

- Considering the two measurements separately

Two independent determinations of r_p

$(\nu_t \to r_\mathrm{p}\ ,\ \nu_s \to r_\mathrm{p})$

Consistent results !!!

Using the 2S-HFS prediction

- Combining the two measurements

Two measurements → determine two parameters

$\nu_t, \nu_s \to \Delta E_\mathrm{L}, \Delta E_\mathrm{HFS} \to \boxed{r_\mathrm{p}, r_\mathrm{Z}}$

$$r_\mathrm{Z} = \int d^3r_1\, d^3r_2\, \rho_E(r_1)\rho_M(r_2)|r_1 - r_2|$$

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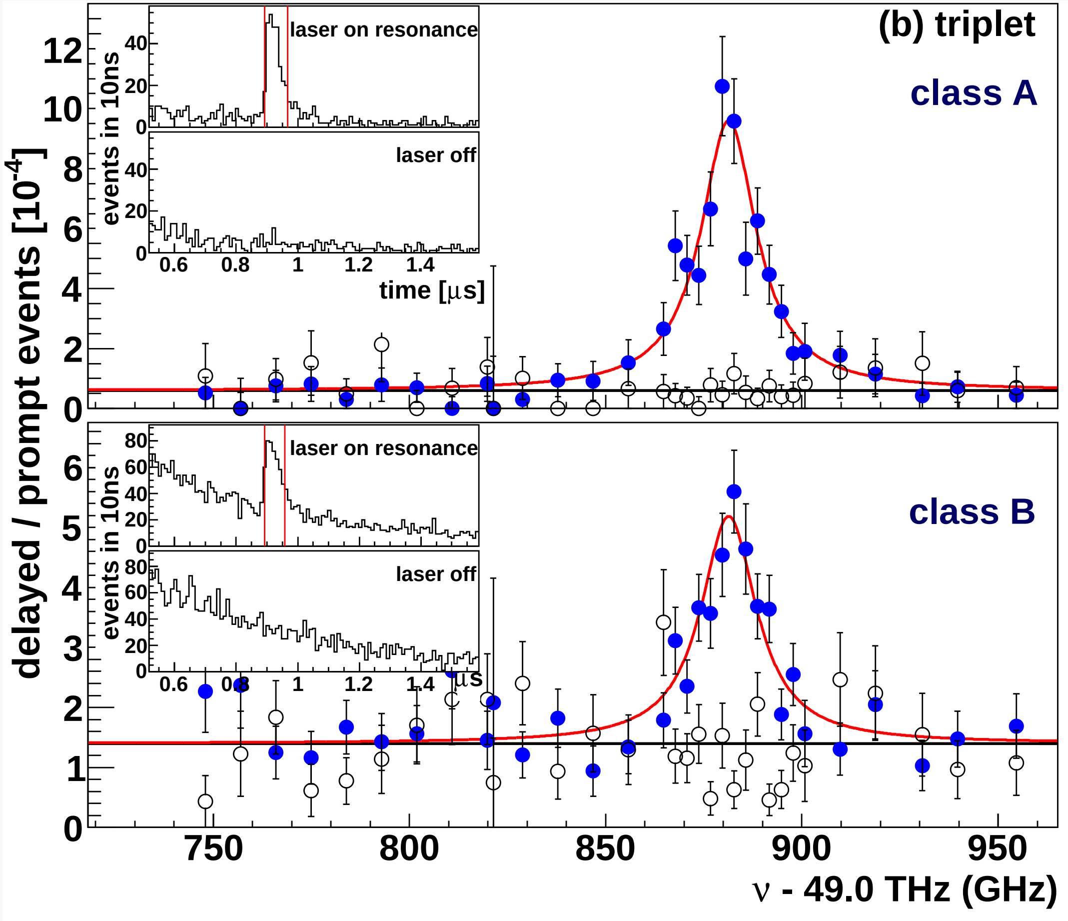
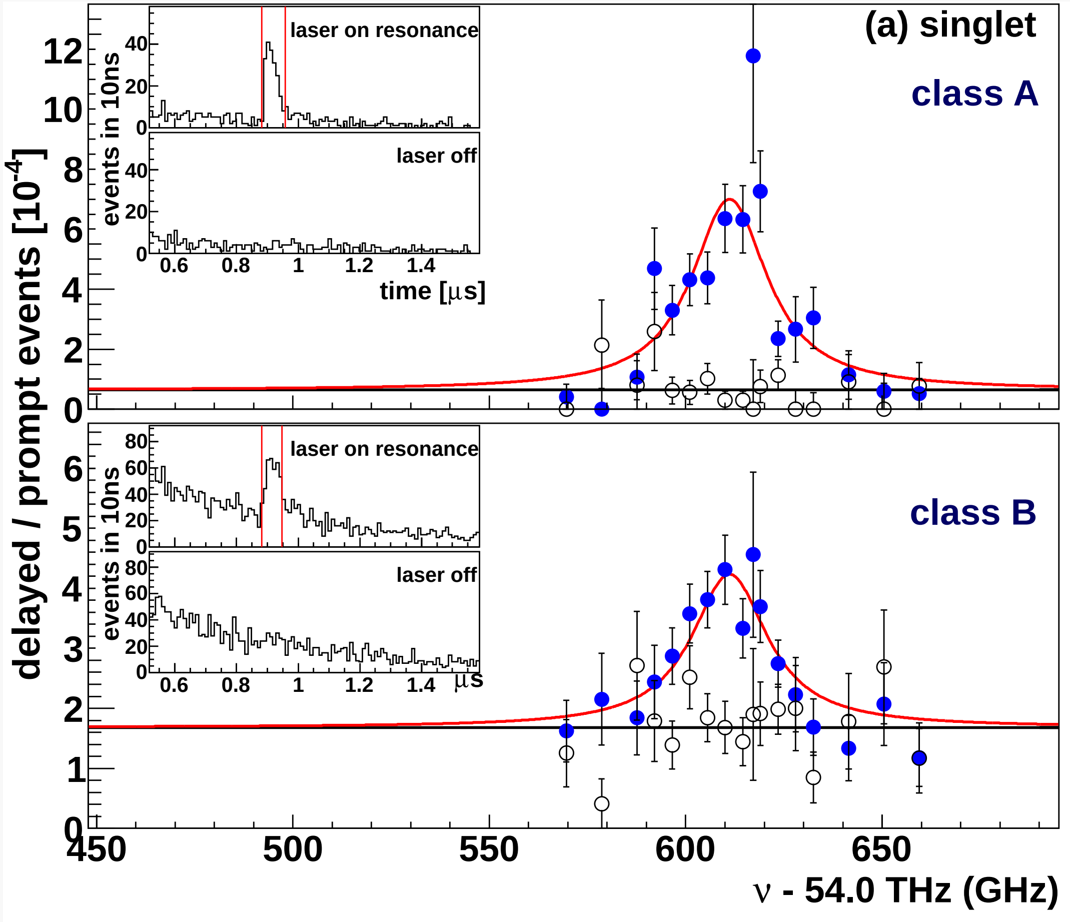
New r_p does NOT dependend on 2S-HFS prediction

We have measured two transitions in μp!

$\nu_t = \nu(2S_{1/2}^{F=1} - 2P_{3/2}^{F=2})$ at $\lambda = 6.0\ \mu\mathrm{m}$

$\nu_s = \nu(2S_{1/2}^{F=0} - 2P_{3/2}^{F=1})$ at $\lambda = 5.5\ \mu\mathrm{m}$





Both resonances are 0.3 meV discrepant from predictions using r_{p} from CODATA

Results on μp: r_p

$$\nu(2S_{1/2}^{F=1} \to 2P_{3/2}^{F=2}) = 49881.88(76)\,\mathrm{GHz}$$

Pohl *et al.*, Nature 466, 213 (2010)

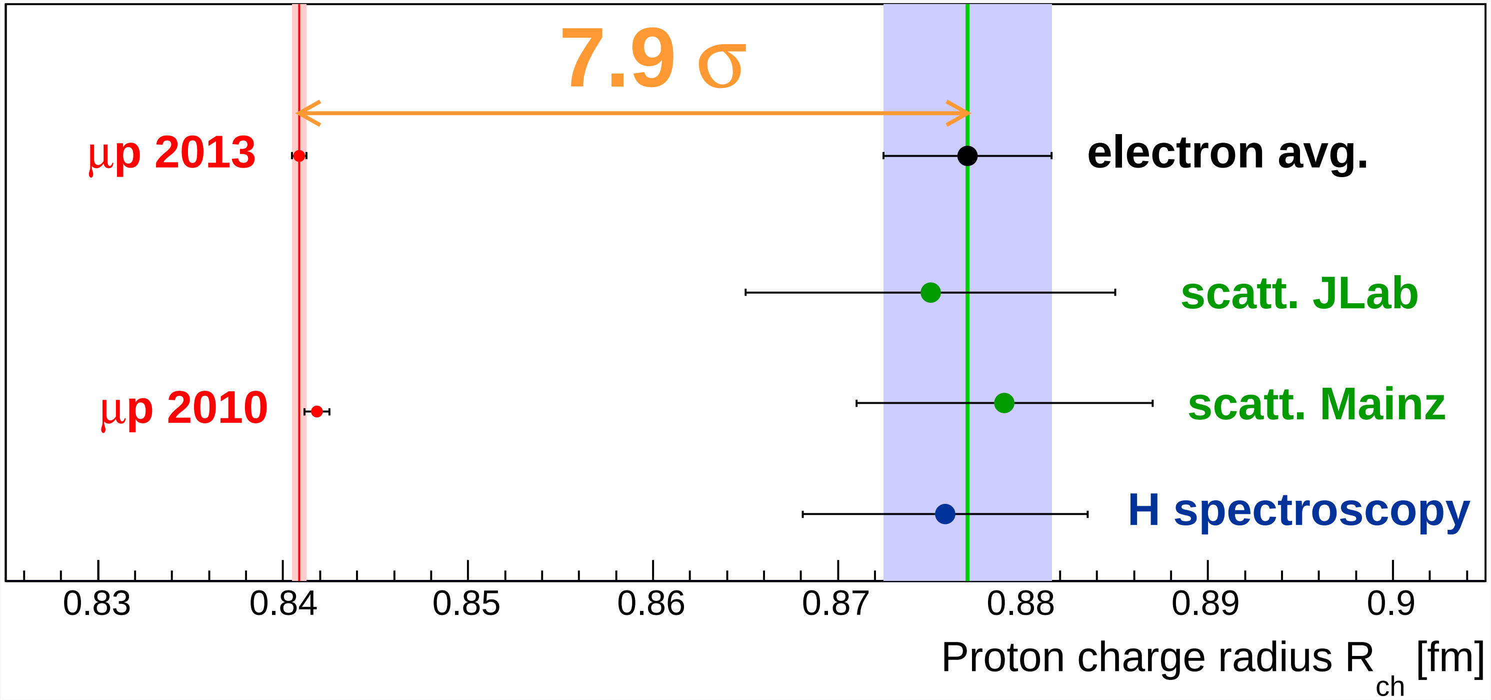
$$49881.35(65)\,\mathrm{GHz}$$

$$\nu(2S_{1/2}^{F=0} \to 2P_{3/2}^{F=1}) = 54611.16(1.05)\,\mathrm{GHz}$$

} Antognini *et al.*, Science 339, 417 (2013)

$\Longrightarrow$ Proton charge radius: $r_\mathrm{p} = 0.84087\ (26)_\mathrm{exp}\ (29)_\mathrm{th} = 0.84087\ (39)\,\mathrm{fm}$

using μp theory summary: Antognini *et al.*, Ann. Phys. 331, 127 (2013) [arXiv:1208.2637]



Proton radius puzzle: What may be wrong?

Bound-state QED?

Proton structure?

Measurements?

Definition p-radius?

“New physics”?

More than 150 publications

Politically correct discussion

Everybody is right!..?

r_p puzzle (1): Is the μp experiment wrong ?

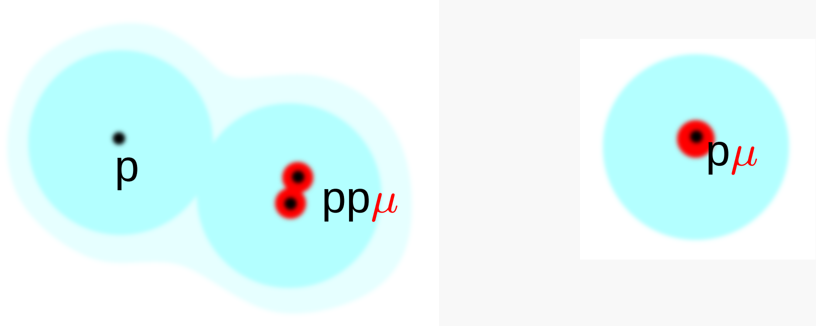
- Systematics?

- laser frequency calibration	300 MHz
- Zeeman effect ($B = 5$ Tesla)	30 MHz
- AC-Stark, DC-Stark shift	< 1 MHz
- Doppler shift	< 1 MHz
- pressure shift (1 mbar)	1 MHz

Systematics shift $\sim 1/m$

Finite size shift $\sim m^3$

- Spectroscopy of $pp\mu$ molecules or $p\mu e$ ions?



Do not exist or too short lived (in 2S state)

Karr and Hilico, PRL 109, 103401 (2012)

Pohl *et al.*, PRL 97, 193402 (2006)

- Frequency mistake by 75 GHz ?

- Huge difference for laser spectroscopy accuracies
- Two ways to calibrate the frequency (consistent)

Discrepancy = 75 GHz $\approx 4\Gamma$

Two consistent μp transition measurements

μp experiment is probably not wrong by 100 σ

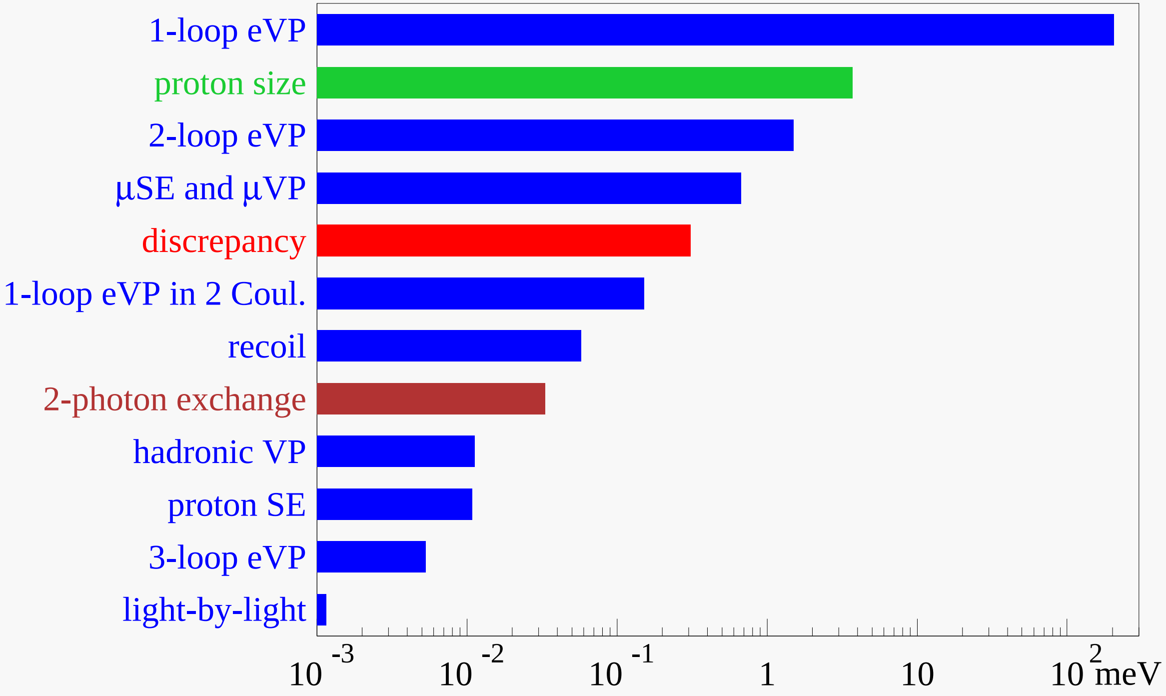
r_p puzzle (2): Is the μp theory wrong ?

Discrepancy = 0.31 meV

Theory uncertainty = 0.0025 meV

$\Rightarrow$ 120δ(theory) deviation?

$$\Delta E^{\mathrm{th}} = 206.0668(25) - 5.2275(10)\, r_\mathrm{p}^2 \;\; [\mathrm{meV}]$$



Pachucki, PRA 60, 3593 (1999)

Borie, arXiv: 1103.1772-v6

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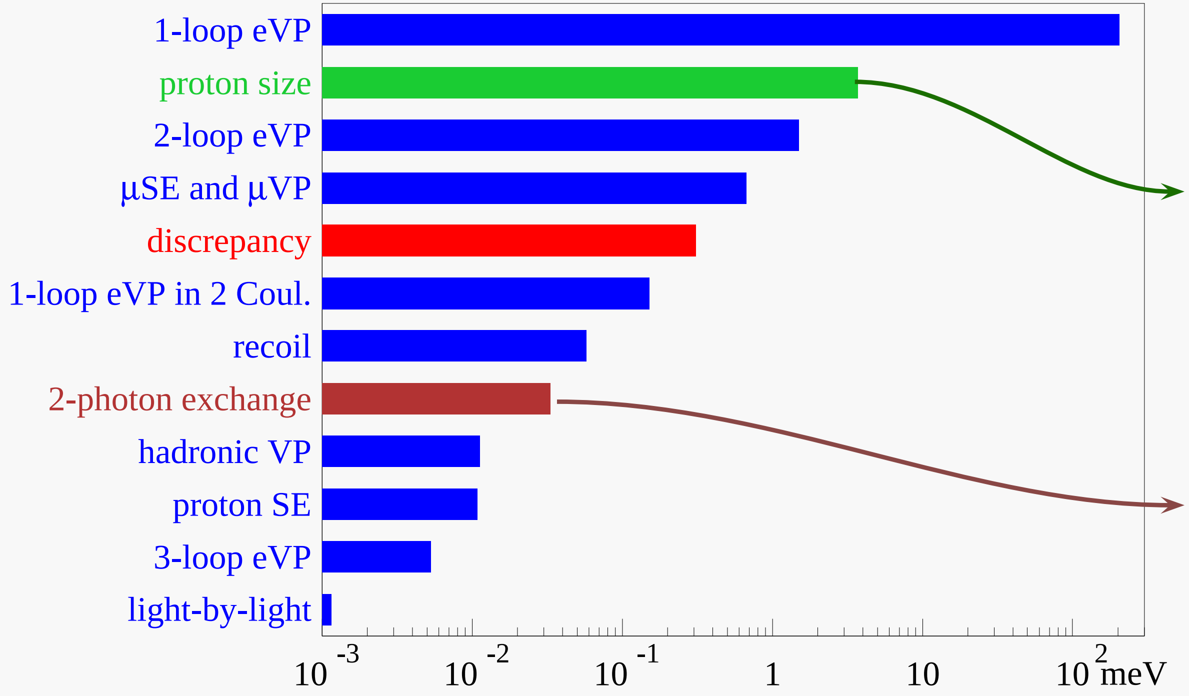

Are one- or two-loop VP wrong?

Proton shape dependence?

Off-shell proton!

Pachucki, PRA 60, 3593 (1999)

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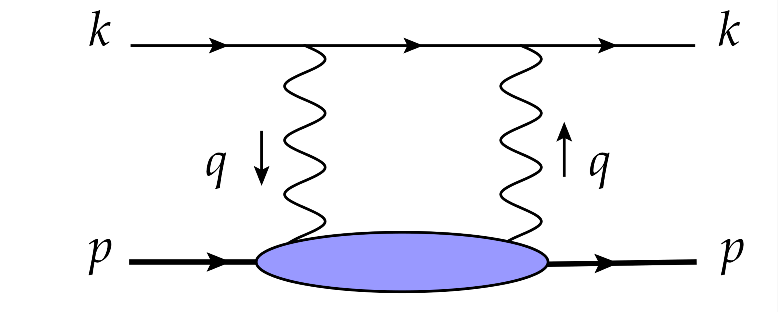

Carlson *et al.*, PRA 84, 020102 (2011)

r_p puzzle (2): Is the μp theory wrong ?

- Can we find a p-shape to solve the discrepancy? DeRujula

YES IF the proton would have charge distributions with very large "tails": $\Delta E_\text{finite size} = \sum_n a_n \langle r_\mathrm{p}^n \rangle$

BUT

bound-state QED expansion	→	a_n decreases rapidly	Friar, Indelicato
e-p scattering data	→	$\langle r_\mathrm{p}^n \rangle$ sufficiently small for $n < 6$	Distler, Miller
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Calculate the TPE contribution via doubly-virtual Compton tensor using dispersion relation.
The imaginary part are the measured proton spin-averaged structure functions

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BUT a subtraction term needed $T_1(0, Q^2)$: at low-Q^2 (NRQED $+\alpha$, β...) and high-Q^2 (QCD) known!
At intermediate-Q^2?

Unknown:	Could be MUCH larger as previosly assumed	Hill *and* Paz, PRL 107, 160402 (2011), Miller
Under control:	Direct calc. of whole contribution in LO χPT	Nevado *and* Pineda, PRC 77, 035202 (2008)
Under control:	χPT expansion to bridge low-Q^2 to high-Q^2	McGovern *and* Birse, EPJA 48 120 (2012)
Under control:	Sum rule + Regge +...photoabsorbtion data	Gorchtein *et al*, PRA 84, 052501 (2013)

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Under control: Direct calc. of whole contribution in LO χPT

$\Delta E_\text{sub} = -0.0042(10)$ meV $\longleftrightarrow$ Discrepancy=0.3 meV

$\Delta E_\text{sub} = -0.0040(5)$ meV

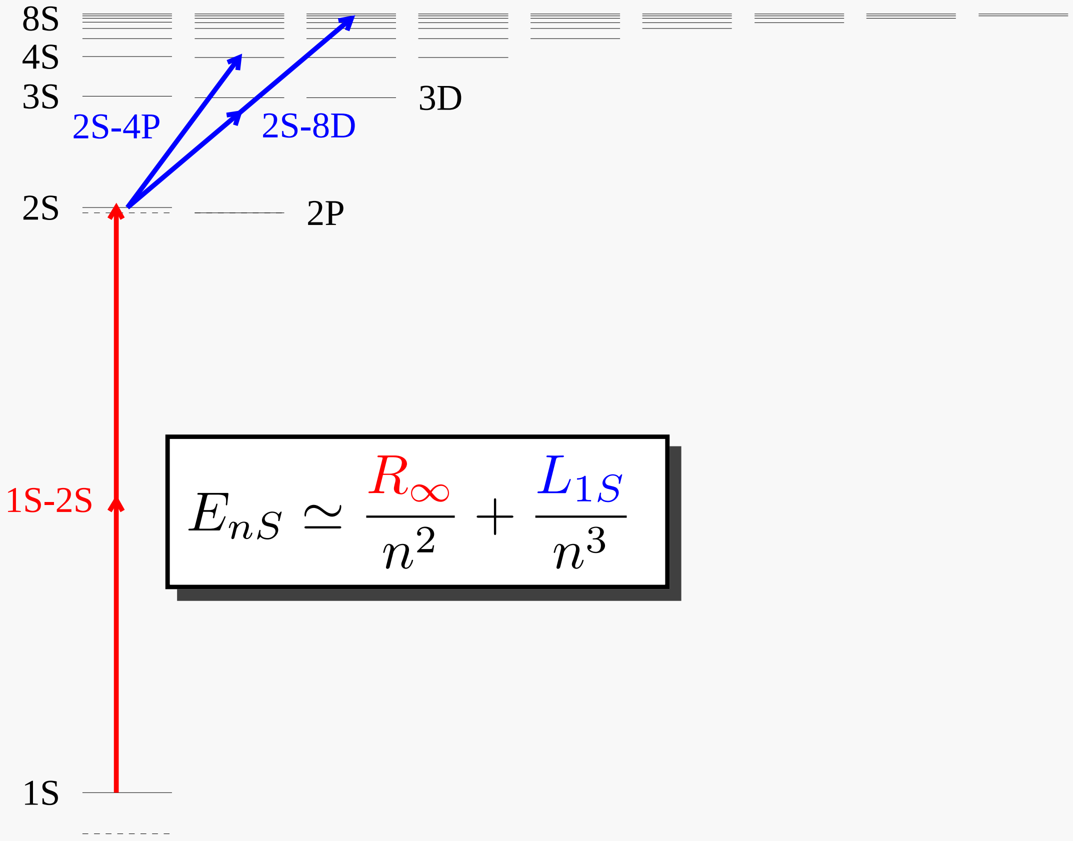
Hill *and* Paz, PRL 107, 160402 (2011), Miller
Nevado *and* Pineda, PRC 77, 035202 (2008)
cGovern *and* Birse, EPJA 48 120 (2012)
rchtein *et al*, PRA 84, 052501 (2013)

r_p puzzle (3): Is H-spectroscopy wrong ?

Two measurements → two unknown: R_∞ and $L_{1S}^{\exp}$

$$\Updownarrow$$

$$L_{1S}^{\mathrm{th}}(r_\mathrm{p}) = 8171.636(4) + 1.5645\, r_\mathrm{p}^2 \quad \mathrm{MHz}$$



$$\boxed{E_{nS} \simeq \frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}}$$

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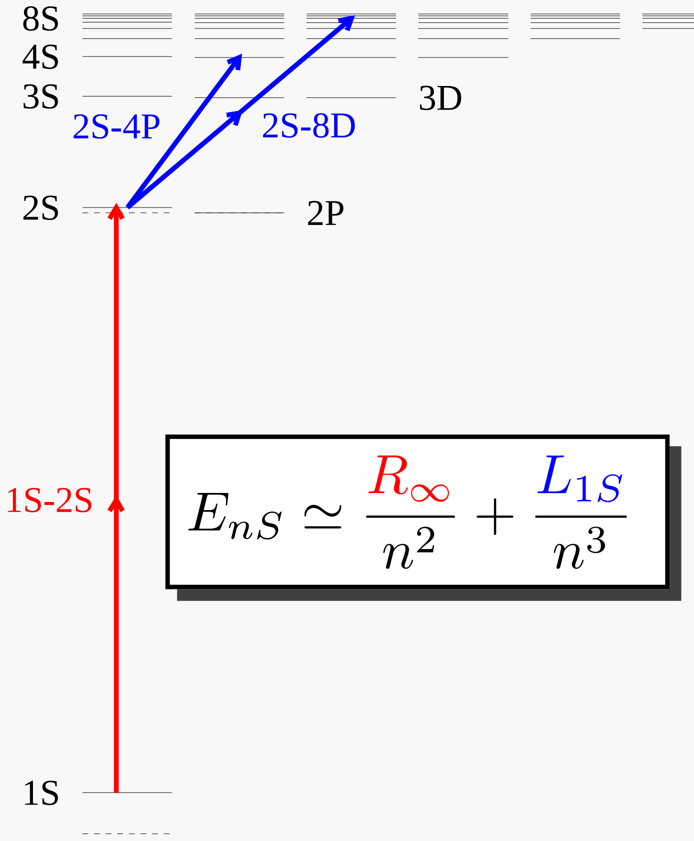

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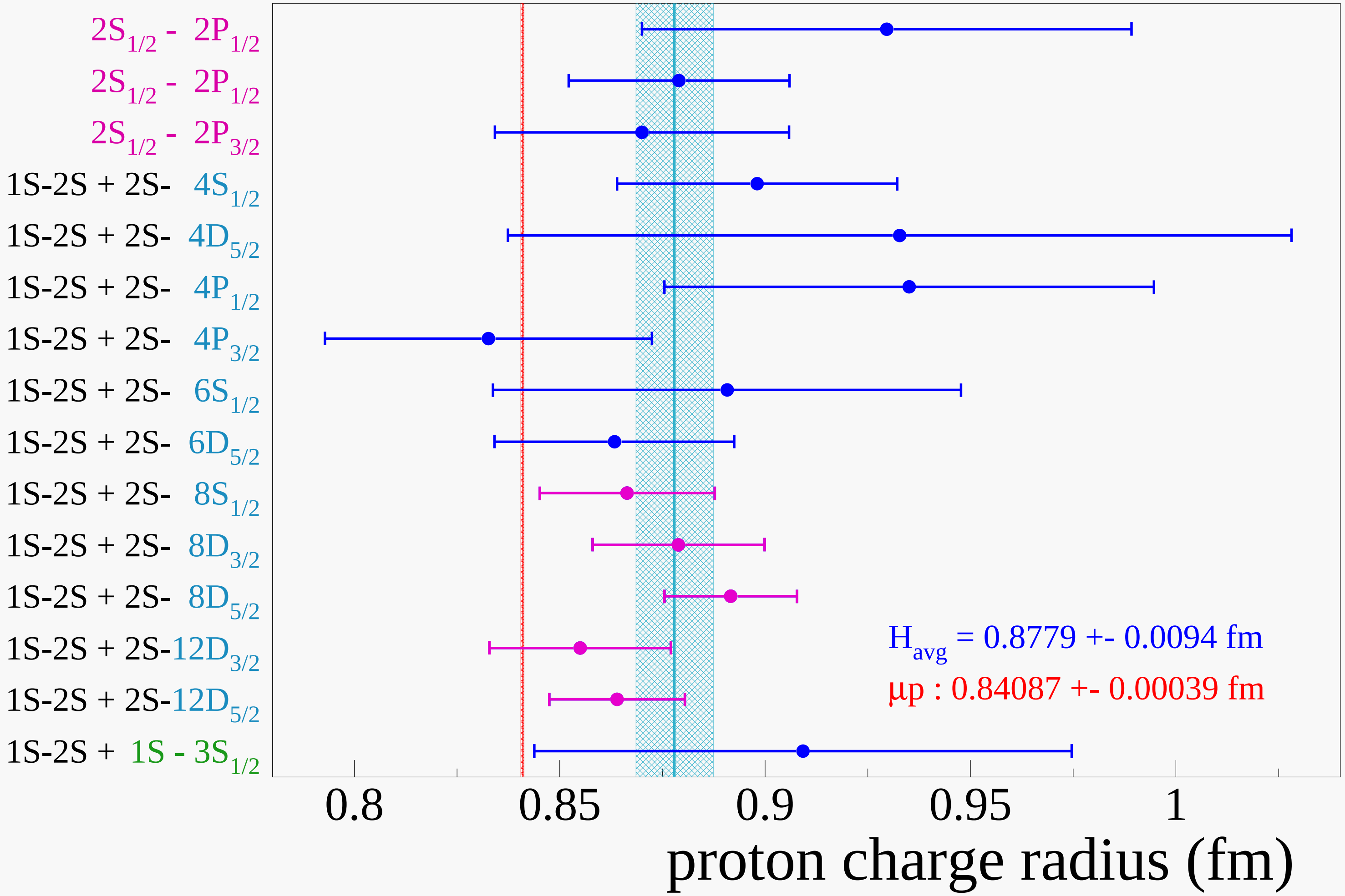

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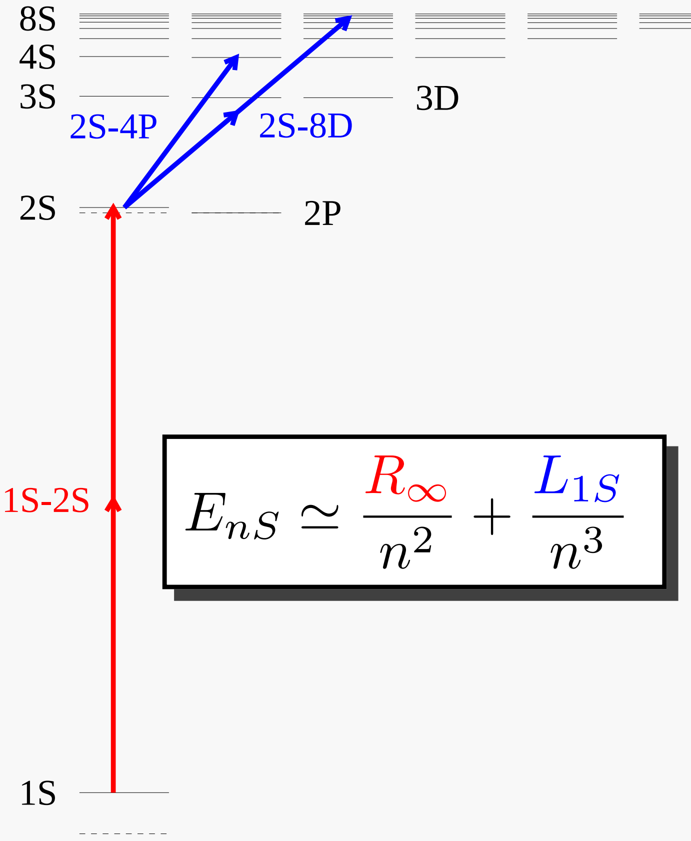

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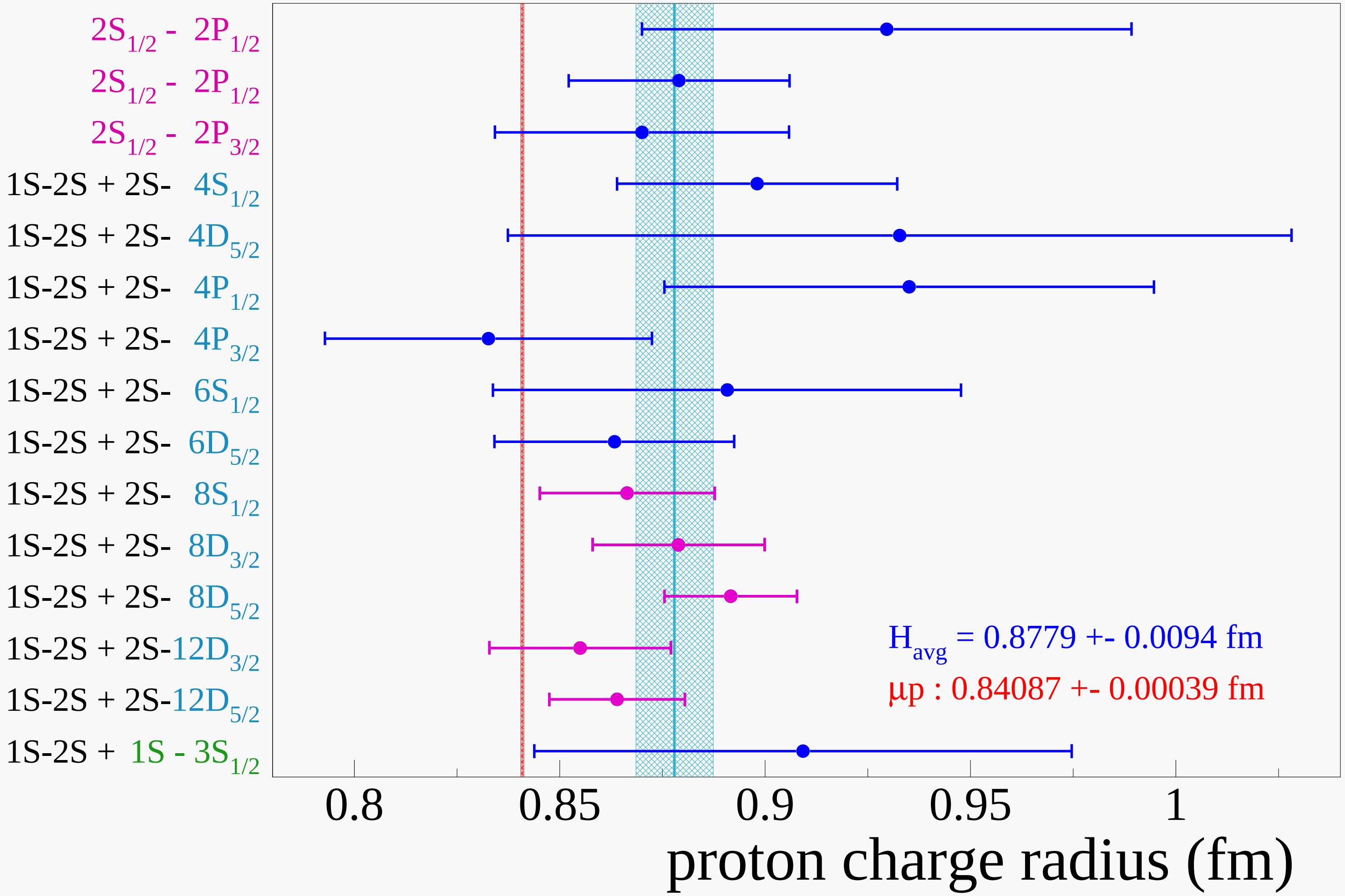
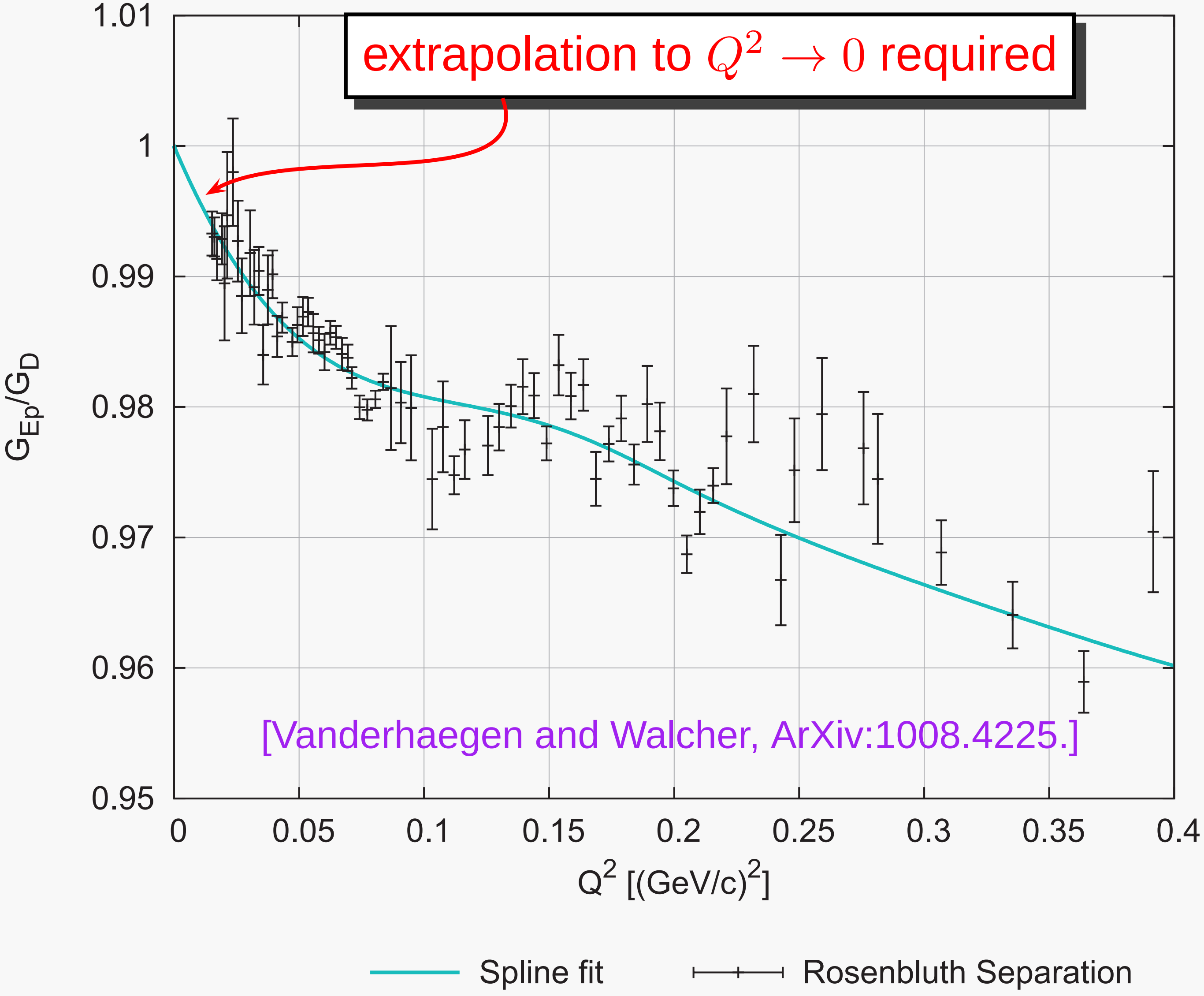


Discrepancy $< 3\sigma$
for individual H meas.

r_p puzzle (5): Is e-p scattering wrong ?

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Ros.}} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \frac{1}{(1+\tau)}\left(\varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2)\right)$$

$$\boxed{\langle r_\mathrm{p}^2\rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\bigg|_{\mathrm{Q^2=0}}}$$



extrapolation to $Q^2 \rightarrow 0$ required

[Vanderhaegen and Walcher, ArXiv:1008.4225.]

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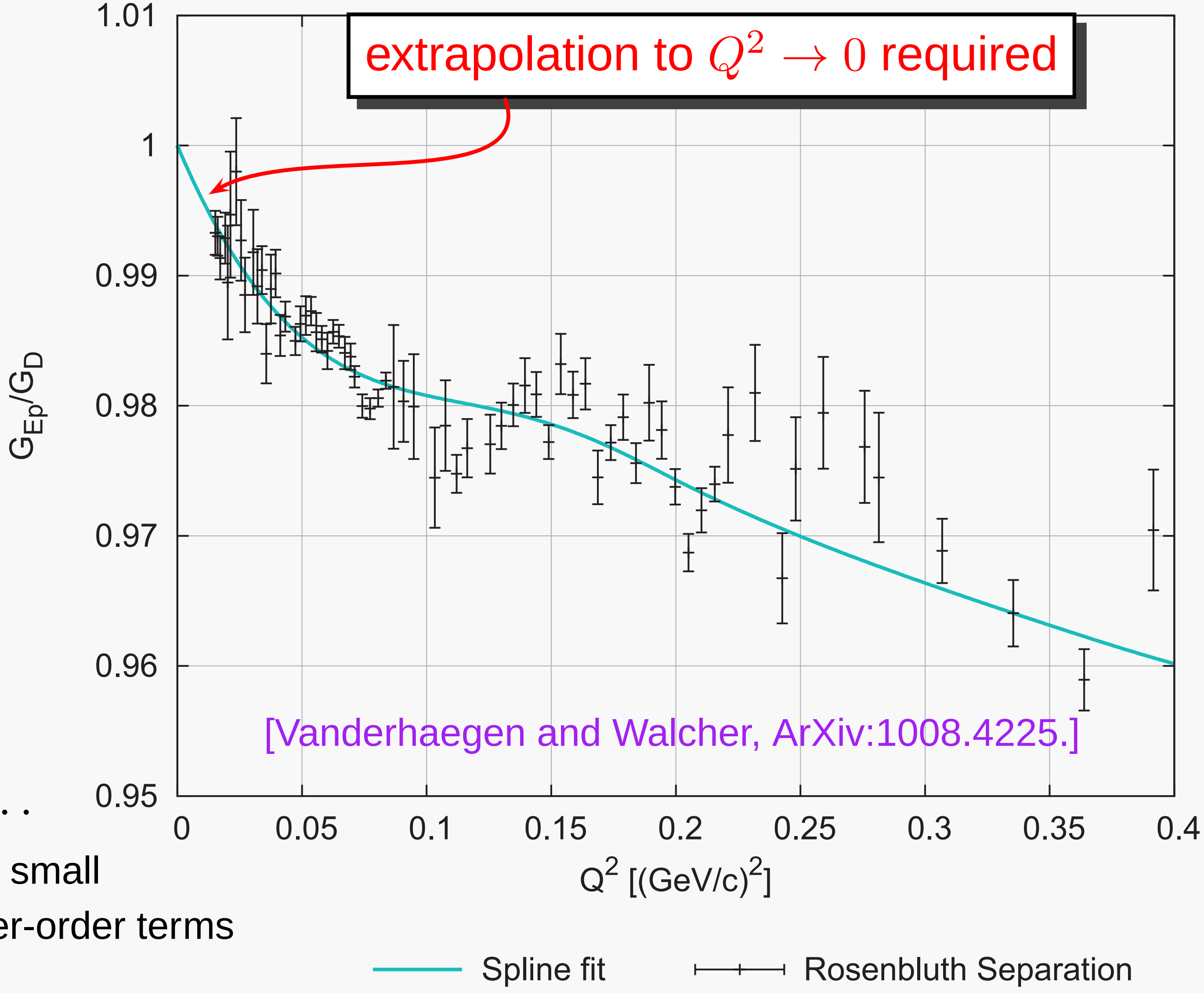
Needs a fit
Model dependence?

Sick, PLB 576, 62 (2003)
Hills and Paz, PRD 82, 113005 (2010)
Bernauer et al, PRL 105, 242001 (2010)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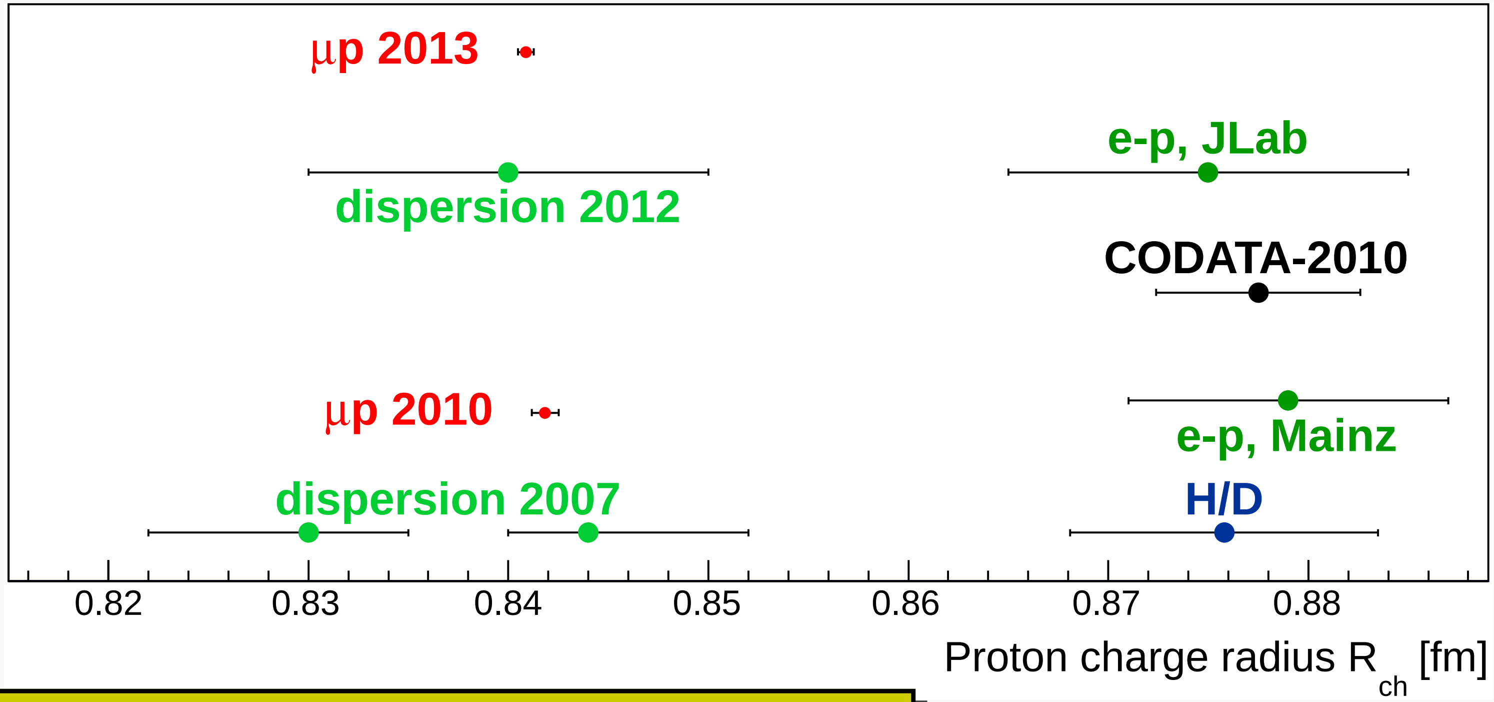
$$G_E(Q^2) = 1 + \frac{Q^2}{6}\langle r_{\mathrm{p}}^2\rangle + \frac{Q^4}{120}\langle r_{\mathrm{p}}^4\rangle + \ldots$$

- Very low Q^2 yields slope but sensitivity is small
- Larger Q^2 more sensitive but larger higher-order terms

Proton charge radii

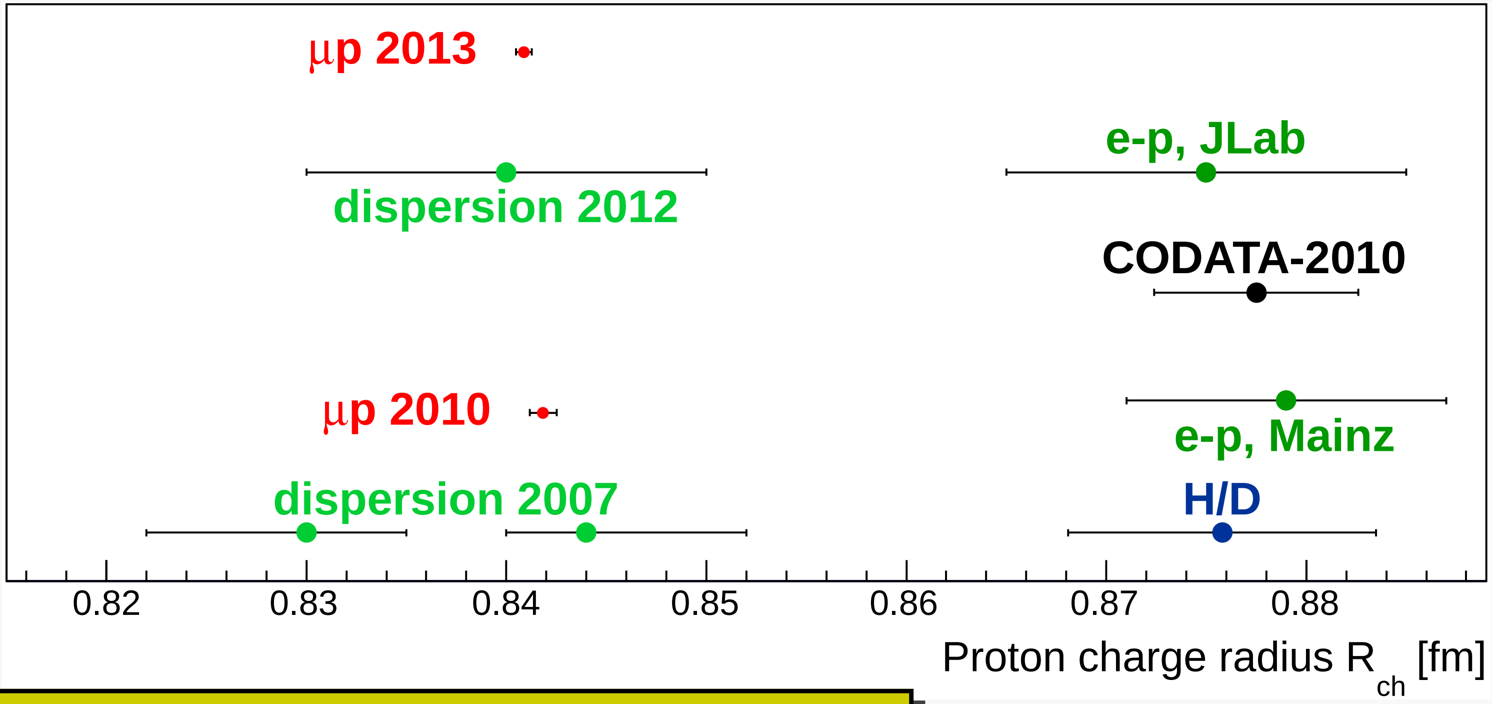


Analysis of e-p, e-n scattering data using VMD and dispersion relations gives radii in agreement with $\mu\mathrm{p}$ albeit a larger χ^2.

Extrapolation of scattering data? R_∞ and higher transitions in H?

The two transition measurements in $\mu\mathrm{p}$ at very different wavelengths are consistent.

Proton charge radii



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BSM physics?

r_p puzzle (6): New physics?

Imply breakdown of muon-electron universality
BUT must evade limitations from other data
(Fix 40’000 ppm discrepancy in $\mu\mathrm{p}$ and 2 ppm discrepancy in a_μ)

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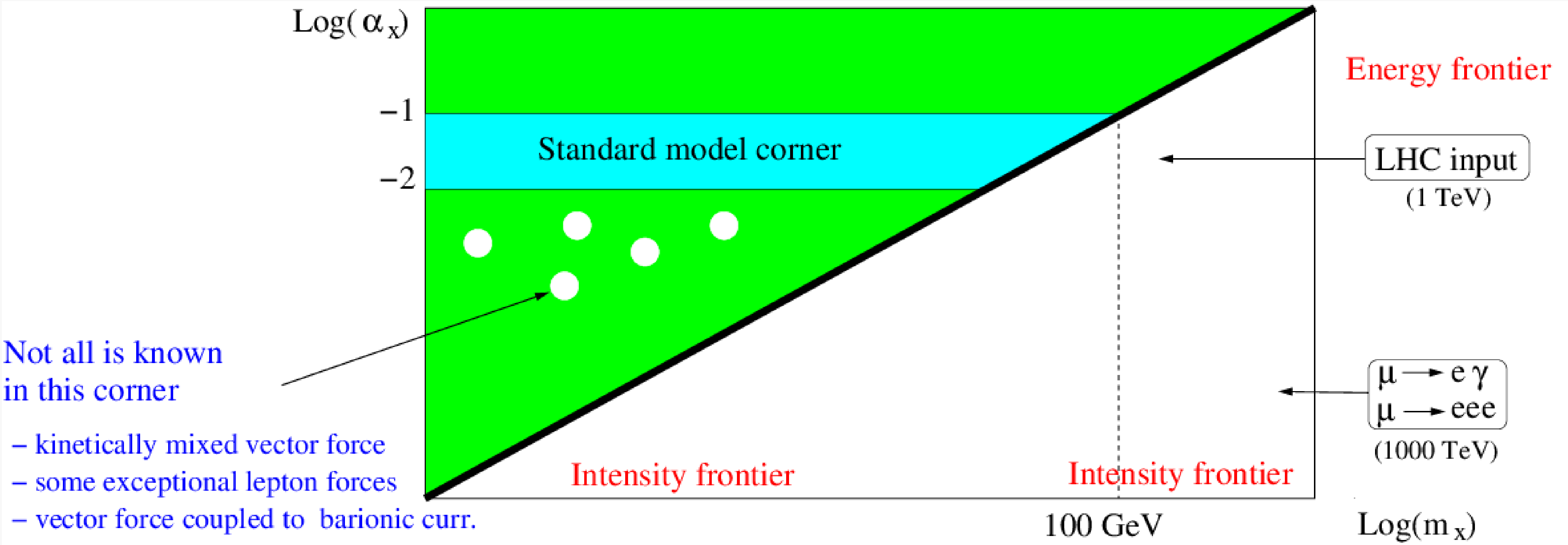

Not all is known
in this corner

- kinetically mixed vector force
- some exceptional lepton forces
- vector force coupled to barionic curr.

[after Pospelov]

If r_p reveals new physics:
$\alpha_x = O(10^4 G_F)$ and $m_x \in [1 - 1000]$ MeV

r_p puzzle (6): New physics?

- Several models have been discussed and discarded because of low energy constraints: $(g-2)_{\mu/e}$, μe, H, $\mu\mathrm{Si}$ spectroscopy, J/Ψ, π, K, η decay widths, n-scattering $\ldots$

- Strange survivors:

  - MeV force carrier coupling only to right-handed muons (parity-violating muonic force).
    Batell, McKeen and Pospelov, PRL 107, 011803 (2011)
  - MeV force carrier with couplings to e and n suppressed relative to couplings to μ and p.
    Trucker-Smith and Yavin, PRD 83, 101702 (2011)
  - MeV new particles, with fine-tuning scalar/pseudoscalar or polar/axial vector and preferential coupling to second-generation. Rislow and Carlson, PRD 86, 035013 (2012)

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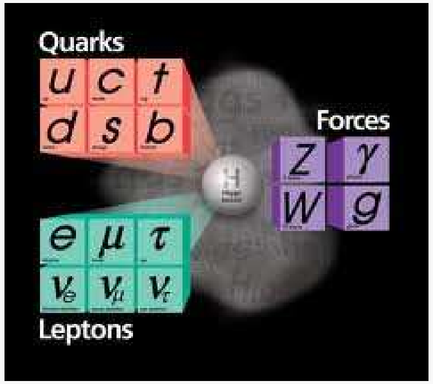
Models exist which escape the many constrains but at “high price”:
targeted coupling and fine tuning

r_p values could be used to constraints a variety of new-physics scenarios

Window for new physics is very small.
BUT more natural extension could come into play if $r_p^{\mathrm{H}} < r_p^{\mu\mathrm{p}} < r_p^{\mathrm{scatt}}$

IF R_∞ will “move”

Motivation, summary, conclusions, outlook



New physics

scattering
$e + p \rightarrow e + p$
$\mu + p \rightarrow \mu + p$
$\gamma + p \rightarrow \gamma + p$
...

Test of H energy levels
Bound-state QED

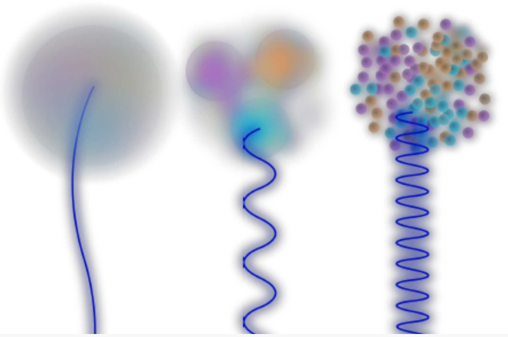
Mu= $\mu^+ e^-$

Ps= $e^+ e^-$

H-spectroscopy

μp and μd

$\mu\,\mathrm{He}^+$

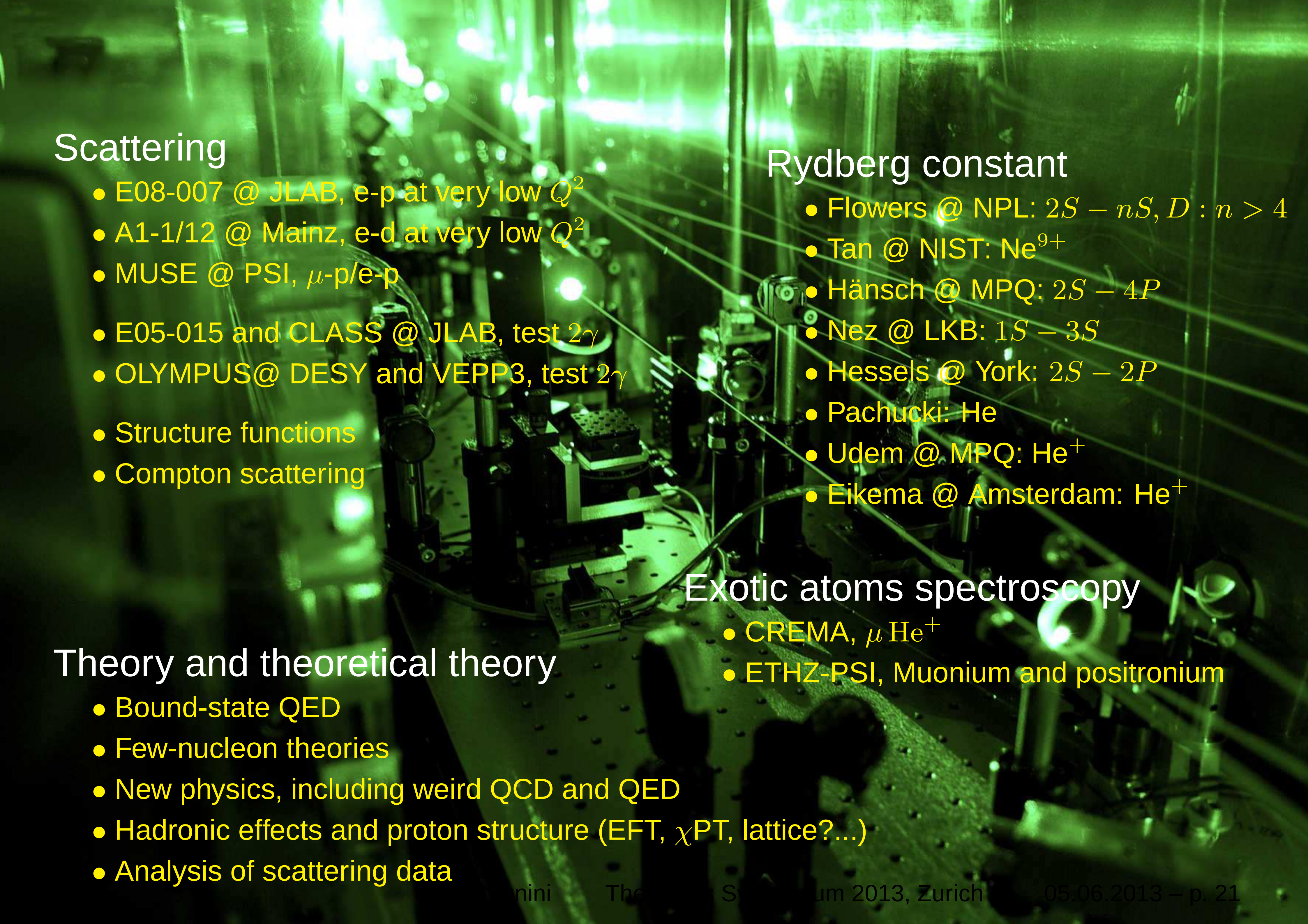


nucl. theories
nucl. potentials

p-structure

EFT, χpt, VMD
Lattice QCD

Proton charge radius
Proton Zemach radius
Deuteron charge radius



Scattering

- E08-007 @ JLAB, e-p at very low Q^2
- A1-1/12 @ Mainz, e-d at very low Q^2
- MUSE @ PSI, μ-p/e-p

- E05-015 and CLASS @ JLAB, test 2γ
- OLYMPUS@ DESY and VEPP3, test 2γ

- Structure functions
- Compton scattering

Rydberg constant

- Flowers @ NPL: $2S - nS, D : n > 4$
- Tan @ NIST: Ne^{9+}
- Hänsch @ MPQ: $2S - 4P$
- Nez @ LKB: $1S - 3S$
- Hessels @ York: $2S - 2P$
- Pachucki: He
- Udem @ MPQ: He^{+}
- Eikema @ Amsterdam: He^{+}

Exotic atoms spectroscopy

- CREMA, $\mu\,\mathrm{He}^{+}$
- ETHZ-PSI, Muonium and positronium

Theory and theoretical theory

- Bound-state QED
- Few-nucleon theories
- New physics, including weird QCD and QED
- Hadronic effects and proton structure (EFT, χPT, lattice?...)
- Analysis of scattering data

CREMA collaboration

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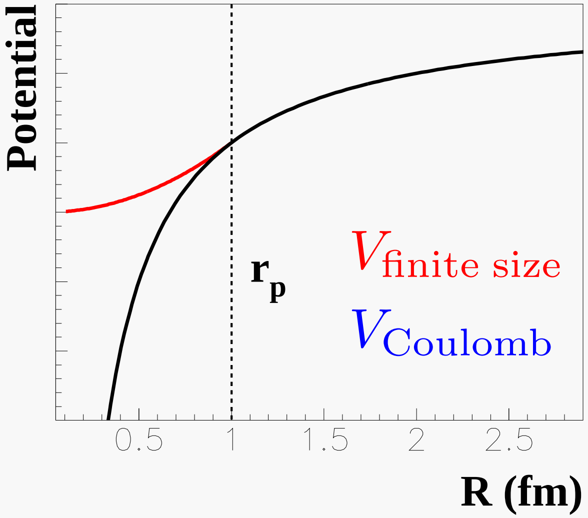
Atomic energy levels and the proton size

$$\Delta E = \Delta E_{\text{QED}} + \Delta E_{\text{fs}}$$

$$\boxed{\begin{aligned} \Delta E^{(0)}_{\text{fs}} &= \frac{2\pi(Z\alpha)}{3} \langle r_{\text{p}}^2 \rangle \, |\Psi_n(0)|^2 \\ &= \frac{2(Z\alpha)^4}{3n^3} m_r^3 \langle r_{\text{p}}^2 \rangle \, \delta_{l0} \end{aligned}}$$

$$\boxed{m_\mu \approx 200 m_e}$$



From $\bar{\nabla} \cdot \bar{E} = 4\pi\rho \rightarrow$ potential V

$$\Delta E^{(0)}_{\text{fs}} = \langle \bar{\Psi} | V_{\text{Coulomb}} - V_{\text{fin.size}} | \Psi \rangle$$

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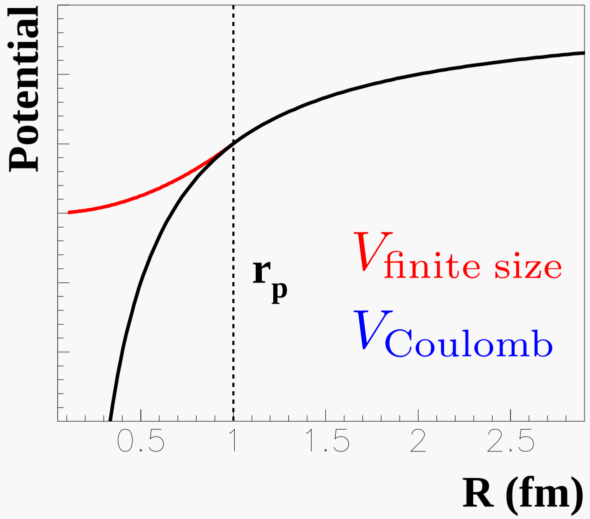


From $\bar{\nabla}\cdot\bar{E} = 4\pi\rho \to$ potential V

$$\Delta E^{(0)}_{\rm fs} = \langle\bar{\Psi}|V_{\rm Coulomb} - V_{\rm fin.size}|\Psi\rangle$$

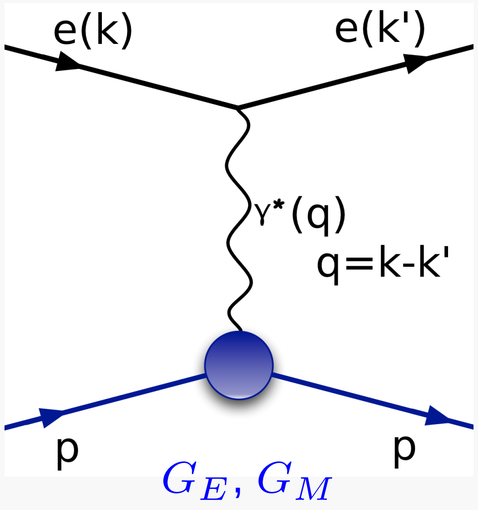
$$G_E(\mathbf{q^2}) = \int d^3r\,\rho_E(\mathbf{r})e^{-i\mathbf{q}\cdot\mathbf{r}} \simeq Z(1 - \frac{\mathbf{q^2}}{6}r_{\rm p}^2 + \cdots)$$

$$\boxed{r_{\rm p}^2 \equiv \int d^3r\,\rho_E(\mathbf{r})r^2}$$

$$\Delta V(r) = -\frac{Z\alpha}{r} - V(r)$$

$$\Delta V(\mathbf{q}) = \frac{4\pi Z\alpha}{\mathbf{q}^2}\left(1 - G_E(\mathbf{q^2})\right) \simeq \frac{2\pi(Z\alpha)}{3}r_{\rm p}^2$$

$$\Delta V(r) = \frac{2\pi(Z\alpha)}{3}\,r_{\rm p}^2\,\delta(r)$$



Results on μp: r_Z

Difference of the two transitions → 2S-HFS in μp: $\Delta E_{\mathrm{HFS}} = 22.8089(51)$ meV

$\Longrightarrow$ Proton Zemach radius: $r_Z = 1.082(31)_{\mathrm{exp}}(20)_{\mathrm{th}} = 1.082(37)$ fm

$$r_Z = \int d^3r_1\, d^3r_2\, \rho_E(r_1)\rho_M(r_2)|r_1 - r_2|$$

Contains information of the magnetic distributions of the proton